Constraining on the CKM Matrix

FPCP - Vancouver - April 12th, 2006

Jérôme Charles (CPT - Marseille)

for the CKMfitter group

Constraînts on the CKM Matrix

le Triangle d’Unitarité sous toutes les coutures

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The Unitarity Triangle

unitarity-exact and convention-independent version of the Wolfenstein parametrization

\[ \lambda^2 \equiv \frac{|V_{us}|^2}{|V_{ud}|^2 + |V_{us}|^2} \quad \Lambda^2 \lambda^4 \equiv \frac{|V_{cb}|^2}{|V_{ud}|^2 + |V_{us}|^2} \]
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\[ \bar{\rho} + i \bar{\eta} \equiv -\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \]

there is no need to stop at \( O(\lambda^4) \)!

\[ (\bar{\rho}, \bar{\eta}) \]

\[ (0, 0) \quad (1, 0) \]
The global CKM fit

uses all constraints on which we think we have a good theoretical control
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$|V_{ud}|, |V_{us}|, |V_{cb}|$  PDG06
The global CKM fit uses all constraints on which we think we have a good theoretical control

| $|V_{ud}|$, $|V_{us}|$, $|V_{cb}|$ | PDG06 |
|--------------------------------|-------|
| $\varepsilon_K$           | exp: KTeV/KLOE, theo: CKM05 | $B_K = 0.79 \pm 0.04 \pm 0.09$ |
| $|V_{ub}|$                  | PDG06 | excl. $(3.94 \pm 0.28 \pm 0.51) \times 10^{-3}$ |
|                             |      | incl. $(4.45 \pm 0.23 \pm 0.39) \times 10^{-3}$ |
| $\Delta m_d$               | exp: last WA, theo: CKM05    | $\xi = 1.24 \pm 0.04 \pm 0.06$ |
| $\Delta m_s$               | exp: you guess, theo: CKM05  | $f_{B_{B_s}}\sqrt{B_s} = 271 \pm 38$ GeV |

note: we have splitted errors into stat. ± theo.
The global CKM fit uses all constraints on which we think we have a good theoretical control

<table>
<thead>
<tr>
<th>Constraint</th>
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<th>Theoretical Sources</th>
<th>Value</th>
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<tbody>
<tr>
<td>$</td>
<td>V_{ud}</td>
<td>,</td>
<td>V_{us}</td>
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$B \to \tau\nu$ exp: last WA, theo: CKM03-05 $f_{B_d} = 190 \pm 25 \pm 9$ MeV

note: we have splitted errors into stat. $\pm$ theo.
More on selected inputs...

the angle $\alpha$

the best constraint comes from the $\rho\rho$ modes; thanks to the BaBar update on $\rho^+\rho^0$ the data are now fully compatible with a closed isospin triangle
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waiting for Belle: Dalitz $\rho \pi$, and $\rho^0 \rho^0$ modes!
...more on selected inputs...

the angle $\gamma$ (preliminary)

the analysis is non trivial:

naive interpretation of $\chi^2$ in terms of the error function underestimates the error on $\gamma$ because of the bias on $\tau_B$ due to $\tau_B > 0$; both Babar and Belle use their own frequentist approach, while we use a different one

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summer
we find a somewhat
looser constraint, with
$\gamma = (62^{+35}_{-25})^\circ$
...more on selected inputs

the oscillation frequency $\Delta m_s$

all details have been given on Sunday (D0) and Tuesday (CDF);
...more on selected inputs

the oscillation frequency $\Delta m_s$

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just look at this plot!
...more on selected inputs

the oscillation frequency $\Delta m_s$

all details have been given on Sunday (D0) and Tuesday (CDF);
just look at this plot!
however, the measured likelihood function has a complicated structure and does not contain enough information to perform a full frequentist analysis.
it would be great to provide us with a Confidence Level curve, or even better, the PDF($\Delta m_{s\text{meas}} \mid \Delta m_{s\text{true}}$)
The global CKM fit: results...

EPS05
all constraints together
The global CKM fit: results...

FPCP06
without $\Delta m_s$ (CDF)
all constraints together
The global CKM fit: results!

FPCP 06
with $\Delta m_s$ (CDF)
all constraints together
Testing the CKM paradigm

CP-conserving...

...vs. CP-violating
Testing the CKM paradigm

CP-conserving...

no angles (with theory)...

...vs. CP-violating

...vs. angles (without theory)
Testing the CKM paradigm

d... vs. loop
Testing the CKM paradigm

The $(\bar{\rho}, \bar{\eta})$ plane is not the whole story, still the overall agreement is impressive!
Theoretical uncertainties...

all non angle measurements uncertainties are now dominated by theory; however a lot of progress in analytical calculations and lattice simulations has been made recently

using traditional approaches

using improved staggered fermions
and theoretical correlations

from Okamoto et al. (2005), splitting into stat. ± theo.

\[ f_{B_d} = 216 \pm 22 \text{ MeV} \]
\[ f_{B_s}/f_{B_d} = 1.20 \pm 0.03 \]
\[ B_{B_d} = 1.257 \pm 0.095 \pm 0.021 \]
\[ B_{B_s} = 1.313 \pm 0.093 \pm 0.014 \]

leads to \( \xi = 1.226 \pm 0.071 \pm 0.033 \) and \( f_{B_d} \sqrt{B_{B_d}} = 242 \pm 26 \pm 2 \text{ MeV} \), while

\[ f_{B_d} = 216 \pm 22 \text{ GeV} \]
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leads to \( \xi = 1.226 \pm 0.035 \pm 0.031 \), \( f_{B_d} \sqrt{B_{B_d}} = 242 \pm 26 \pm 9 \text{ MeV} \) and \( B_{B_d} = 1.258 \pm 0.094 \pm 0.045 \).
$|\sin(2\beta + \gamma)|$

from $b \to c\bar{u}d, u\bar{c}d$
Selected fit predictions

the Wolfenstein parameters

\[ \lambda = 0.2272^{+0.0010}_{-0.0010} \quad \Lambda = 0.809^{+0.014}_{-0.014} \]

\[ \bar{\rho} = 0.197^{+0.026}_{-0.030} \quad \bar{\eta} = 0.339^{+0.019}_{-0.018} \]

the Jarlskog invariant

\[ J = (3.05 \pm 0.18) \times 10^{-5} \]

the UT angles

\[ \alpha = (97.3^{+4.5}_{-5.0})^\circ \quad \beta = (22.86^{+1.00}_{-1.00})^\circ \quad \gamma = (59.8^{+4.9}_{-4.1})^\circ \]

\( B_s - \bar{B}_s \) mixing

\[ \Delta m_s = 17.34^{+0.49}_{-0.20} \text{ ps}^{-1} \]

\( B \) leptonic decay

\[ B(B \to \tau \nu) = (9.7 \pm 1.3) \times 10^{-5} \]
New Physics in mixing

model-independent parametrization

\[
\langle B_q \mid \mathcal{H}^{SM+NP}_{\Delta B=2} \mid \bar{B}_q \rangle \equiv \langle B_q \mid \mathcal{H}^{SM}_{\Delta B=2} \mid \bar{B}_q \rangle \times (1 + h_q e^{2i\sigma_q})
\]
assuming $\Delta m_s = 20.000 \pm 0.011 \text{ ps}^{-1}$ and $\sin 2\beta_s = 0.036 \pm 0.028$ (one year LHCb running)
Constraint on supersymmetric charged Higgs from $B \to \tau \nu$

![Graph showing the constraint on supersymmetric charged Higgs from $B \to \tau \nu$. The graph plots $m_{H^+}$ (GeV/c^2) against $\tan \beta$ with a color gradient indicating the confidence level (CL).]
most of SU(3)-based analyses of charmless $B \to \pi\pi$, $K\pi$, $K\bar{K}$ decays neglect annihilation/exchange topologies
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this assumption is not mandatory!

"$\alpha$" from $B \to \pi^+\pi^-, K^+\pi^-, K^+K^-$
The Unitarity Triangle from flavor SU(3)


most of SU(3)-based analyses of charmless $B \to \pi\pi, K\pi, K\bar{K}$ decays neglect annihilation/exchange topologies

this assumption is not necessary!

"$\beta$" from $B \to K_S\pi^0, \pi^0\pi^0, K^+K^-$
using all $B \rightarrow PP$ observables (today)
using all $B \to PP$ observables (today $\to$ tomorrow)
Depuzzling $B \rightarrow K\pi$

using $(\bar{\rho}, \bar{\eta})_{SM}$ and all $B \rightarrow PP$ observables, except $BR(B \rightarrow K^+\pi^-)$, $BR(B \rightarrow K^0\pi^0)$ and $S(K_S\pi^0)$

$$R_n = \frac{BR(K^+\pi^-)}{2BR(K^0\pi^0)}$$
Depuzzling $B \to K\pi$

using $(\tilde{\rho}, \tilde{\eta})_{SM}$ and all $B \to PP$ observables, except $\text{BR}(B \to K^+\pi^-)$, $\text{BR}(B \to K^0\pi^0)$ and $S(K_S\pi^0)$

$$R_{\pi} = \frac{\text{BR}(K^+\pi^-)}{2\text{BR}(K^0\pi^0)}$$

\(a \sim 2\sigma\) effect!
Conclusion

congratulations to
Conclusion

congratulations to

BaBar ?...
congratulations to

BaBar ?…

Belle ?…
Conclusion

congratulations to

BaBar ?…
Belle ?…
DO ?…
Conclusion

congratulations to

BaBar ?…
Belle ?…
D0 ?…
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…to Standard Model of course
backup
The CKM movie

Δm_d

Δm_s & Δm_d

|V_{ub}|/|V_{cb}|

\alpha

\gamma

β

excluded area has CL > 0.95
The CKM movie

The figure shows a contour plot with various regions labeled. The plot includes the regions 

- $\Delta m_d$
- $\Delta m_s \& \Delta m_d$
- $\Delta m_d$
- $\varepsilon_K$
- $|V_{ub}/V_{cb}|$
- $\alpha$
- $\gamma$
- $\beta$

The excluded area has CL $> 0.95$.
The CKM movie

- $|V_{ub}/V_{cb}|$
- $\sin 2\beta$
- $\Delta m_d$
- $\Delta m_s \& \Delta m_d$
- $\varepsilon_K$
- $\rho$

Excluded area has CL $> 0.95$

Solution with $\cos 2\beta < 0$ (excl. at CL $> 0.95$)

CKM fit FPCP 06
The CKM movie

- $\sin 2\beta$
- $\Delta m_d$
- $\Delta m_s & \Delta m_d$
- $|V_{ub}/V_{cb}|$
- $\varepsilon_K$
- $\alpha$
- $\gamma$
- $\beta$
- $\rho$

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CKM fit: FPCP 06
The statistical method to extract $\gamma$

The observables depend on $\gamma$ and $\mu$ where $\mu = (r_B, \delta)$

1. minimize $\chi^2(\gamma, \mu)$ with respect to $\mu$ and subtract the minimum $\rightarrow \Delta\chi^2(\gamma)$

2. assume that the true value of $\mu$ is $\mu_t$ $\rightarrow$ PDF$[\Delta\chi^2(\gamma) \mid \gamma, \mu_t]$

3. compute $(1 - CL)_{\mu_t}(\gamma)$ via toy Monte-Carlo

4. maximize with respect to $\mu_t$ $\rightarrow (1 - CL)(\gamma)$

This is a quite general, but very expensive, procedure; coverage must be checked before we assumed that $\mu_t$ was given by the value that minimizes $\chi^2(\gamma, \mu)$ on the real data: studies have shown us that this can lead to an underestimate of the error