QCD Factorization and SCET at NLO

Alex Williamson

Carnegie Mellon University

Outline

- QCD Factorization
  - Description
  - Comparison to SCET approach
- Recent Phenomenological Results
  - $B \to VV$ (a little bit)
- NLO in α_s and $1/m_b$
  - What is necessary at NLO?
  - What has been done so far?
  - Recent α_s^2 corrections
- Conclusions

QCD Factorization

Formalism

Beneke, Buchalla, Neubert, Sachrajda 1999, 2000, 2001
Bauer, Fleming, Pirjol, Stewart 2001

Description of Hadronic B decays

- QCD Factorization is the forerunner of SCET
- calculations in the QCD Factorization approach follow the SCET framework
- there remain differences in approach
- describe two-body B decays in the heavy quark limit
- separate long and short distance physics
- all strong phases treated as calculable
- follow the conventions of QCDF

Formalism (cont.)

$$\langle M_1 M_2 | O | \bar{B} \rangle = F^{B\to M_1} T^I * f_{M_2}\Phi_{M_2} + M_1 \leftrightarrow M_2$$
$$+ T^{II} * f_B \Phi_B * f_{M_1}\Phi_{M_1} * f_{M_2}\Phi_{M_2} + \mathcal{O}(1/m_b)$$

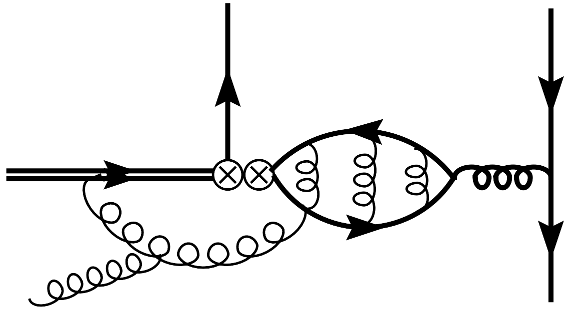
- T^I and T^{II} depend on the process $\bar{B} \to M_1 M_2$ being consider but are perturbatively calculable as an expansion in α_s.
- f_M, f_B, Φ_M, Φ_B are universal and do not depend on the decay process. Unfortunately must be found from QCD sum rules, lattice or experiment.
- QCDF predicts the amplitudes/observables without making use of $B \to MM$ decays (almost)

Differences of opinion...

- Perturbative order of calculations (NLO vs. NNLO)
- QCDF uses input from external sources
- SCET uses some inputs from external sources but also input from $B \to MM$ decays
- QCDF includes a subset of order $1/m_b$ corrections it deems to be large.
- There are parameters in the QCDF approach that are somewhat uncertain: λ_B
  - Experiment may influence value of λ_B
- Treatment of charming penguins

Charming penguins

- since $2m_c \sim m_b$ there are configurations with almost on-shell charm quarks



- QCDF Approach: charming penguins factorize

- SCET Approach: nonpert., NRQCD counting $\alpha_S(2m_c)f(2m_c/m_b)v$
  - more conservative - introduce new nonpert. parameters $A_{cc}^{M_1 M_2}$ that are fit from data

Phenomenology

Beneke, Neubert 2003

- QCDF approach has increased predictive power over the SCET approach of BPRS, but less conservative.
- QCDF had been used to be predict the branching ratios and CP asymmetries of a large number of 2-body charmless modes, $B \to PP$ and $B \to PV$
- recently first predictions for $B \to VV$

B to VV

Beneke, Rohrer, Yang 2005

- identify operators that m_b/λ enhanced over the leading terms, but are also $\alpha_{\rm em}$ suppressed $\implies$ numerically small

$$Q_{7\gamma}^{\mp} = -\frac{e\bar{m}_b}{8\pi^2}\,\bar{D}\sigma_{\mu\nu}(1\pm\gamma_5)F^{\mu\nu}b$$

- cannot explain the puzzle of large transverse polarizations in $K^*\rho$, $K^*\phi$:
- naive factorization:

$$A_0 : A_- : A_+ = 1 : \frac{\Lambda}{m_b} : \left(\frac{\Lambda}{m_b}\right)^2$$

B to VV results

- can demonstrate the large effect of this new operator
- consider ratio of CP-averaged helicity-decay rates

$$S_h = \frac{2\bar{\Gamma}_h(\rho^0 \bar{K}^{*0})}{\bar{\Gamma}_h(\rho^- \bar{K}^{*0})} = \left| 1 - \frac{P_h^{EW}}{P_h} \right| + \Delta_h$$

- determine $|P_h|$ from Br and f_L for $\rho^- \bar{K}^{*0}$ since $A_h(\rho^- \bar{K}^{*0}) = P_h$.

$$\Longrightarrow \; S_- = 1.5 \pm 0.2 \;\; [0.7 \pm 0.1]$$

B to VV results

- could consider the ratio

$$S'_h = \frac{2\bar{\Gamma}_h(\rho^0 \bar{K}^{*-})}{\bar{\Gamma}_h(\rho^- \bar{K}^{*0})} = \left|1 - \frac{P_h^{EW}}{P_h}\right| + \Delta'_h$$

- determine $|P_h|$ from Br and f_L for $\rho^- \bar{K}^{*0}$ as before.

$$\Longrightarrow \; S_- = 0.5 \pm 0.1 \;\; [1.2 \pm 0.1]$$

- in terms of the ratio $f'_h \equiv f_h(\rho^0 \bar{K}^{*-})/f_h(\rho^- \bar{K}^{*0})$,

$$f'_0 = 1.3 \pm 0.1 \;\; [1.1 \pm 0.1] \qquad (\text{exp.} = 1.45^{+0.64}_{-0.58})$$
$$f'_- = 0.4 \pm 0.1 \;\; [0.8 \pm 0.1] \qquad (\text{exp.} = 0.12^{+0.44}_{-0.11})$$

NLO in α_s and $1/m_b$

Motivational modes

- to reduce overall uncertainties
- CP asymmetries will receive large corrections, if at LO the "tree" amplitudes are not coming from $O_{1,2}$ operators
  - $\mathcal{A}^{CP}$ for $B_s^0 \to \pi^0 \eta^{(\prime)}$, $B^- \to \bar{K}^0 \pi^-$ (bad)
  - $\mathcal{A}^{CP}$ for $B^- \to K^0 K^-$, $\bar{B}^0 \to K^0 \bar{K}^0$ (really bad)
- Br for these modes:
  - ($\Delta S = 1$): small corrections expected since "tree" CKM suppressed
  - ($\Delta S = 0$): large corrections expected since "tree" not CKM suppressed
- Need both $1/m_b$ and $\alpha_s(m_b)$ corrections

What is necessary at NLO?

Start with the $1/m_b$ corrections.

- typical corrections of the order $\sim 20\%$.
- SCET is the theoretical framework necessary to describe the $1/m_b$ corrections, but almost no $1/m_b$ corrections have been considered.
- exceptions include the annihilation contributions and chirally enhanced contributions that have been taken into account by BBNS.
- factorization has not been demonstrated at subleading order in $1/m_b$
- there is much to do here.

What about α_s corrections?

- rewrite the basic QCDF formula, as is done in SCET, by writing $T^{II} = H^{II} * J$,

$$\langle M_1 M_2 | O | \bar{B} \rangle = (F^{B\to M_1} T^I + \Xi^{B\to M_1} * H^{II}) * f_{M_2} \Phi_{M_2} + M_1 \leftrightarrow M_2$$

where

$$\Xi^{B\to M_1} \equiv J * f_B \Phi_B * f_{M_1} \Phi_{M_1}$$

- T^I and T^{II} are hard coefficients, calculable in $\alpha_s(m_b) \sim 0.2$ where perturbation theory is expected to be valid. They have the general form $C + \mathcal{O}(\alpha_s(m_b))$.
- Jet function, J, is perturbatively calculable in $\alpha_s(\sqrt{\Lambda_{\rm QCD} m_b})$. J begins at $\mathcal{O}(\alpha_s(\sqrt{\Lambda_{\rm QCD} m_b}))$, can be related to other processes.

Status of NLO corrections

Hill, Becher, Lee, Neubert 2004
Becher, Hill 2004

- As of FPCP 2004
  - T^I was known to $\mathcal{O}(\alpha_s^2(m_b)\beta_0)$ (NLO+),
  - H^{II} was known to $\mathcal{O}(\alpha_s^0(m_b))$ (LO),
  - J was known to $\mathcal{O}(\alpha_s^2(\sqrt{\Lambda_{\mathrm{QCD}} m_b}))$ (NLO).

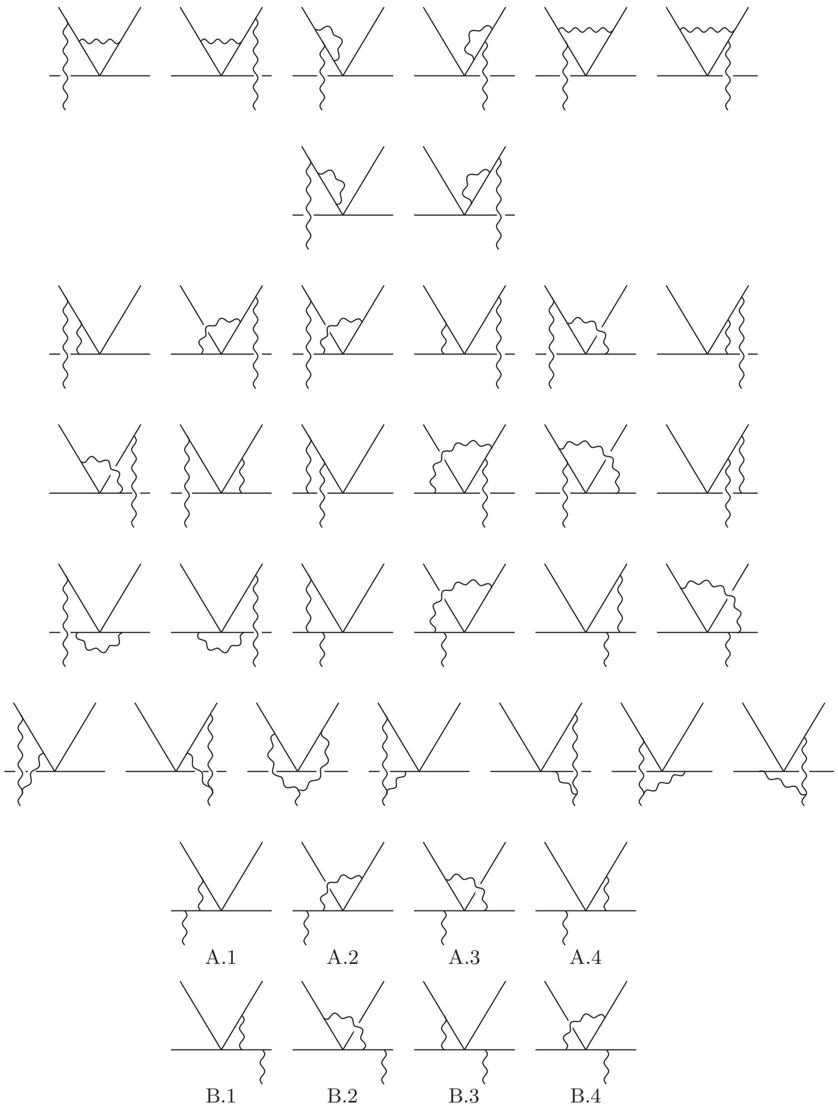
  The α_s^2 corrections to J had been calculated for the purpose of the form factors.

- Recently...
  - part of H^{II} was calculated to $\mathcal{O}(\alpha_s(m_b))$ (NLO)

Perturbative corrections

Beneke, Jäger 2005

- hard matching for H^{II} calculated for tree operators $O_{1,2}$
- hard matching for penguin operators still missing
- taken with earlier results for J, now $\Xi^{B\to M_1}$ is NLO (SCET), NNLO (BBNS)



Perturbative corrections

Hill, Becher, Lee, Neubert 2004
Becher, Hill 2004
Kirlin 2005
Beneke, Yang 2005
Beneke, Jäger 2005

- hard scattering NLO corrections particularly important because they introduce new strong phases.

color allowed:

$$\alpha_1^{\pi\pi} = 1.001 + [0.025 + 0.012i\,]_V - \,[0.024 + 0.020i\,]_{\rm NLO} - [0.009]_{\rm tw3}$$
$$= 0.992^{+0.029}_{-0.054} + (-0.007^{+0.018}_{-0.035})\,i,$$

color suppressed:

$$\alpha_2^{\pi\pi} = 0.272 - [0.152 + 0.077i\,]_V + \,[0.029 + 0.034i\,]_{\rm NLO} + [0.056]_{\rm tw3}$$
$$= 0.205^{+0.171}_{-0.110} + (-0.043^{+0.083}_{-0.065})\,i.$$

Choice of input parameters

- Reexamine how well theory reproduces experiment. With these numbers could compare to $\pi\pi$ branching ratios.
- In trying to reproduce the $\pi\pi$ branching ratios, data favours inputs where

$$|V_{ub}|f_+^{B\pi}(0) = 8.10 \times 10^{-4} = 0.775 \left[|V_{ub}|f_+^{B\pi}(0)\right]_{\rm def}$$

$$\frac{f_\pi f_B}{m_b f_+^{B\pi}(0)\lambda_B} = 1.96 \left[\frac{f_\pi f_B}{m_b f_+^{B\pi}(0)\lambda_B}\right]_{\rm def}$$

- This is consistent with smaller values of the form-factor $f_+^{B\pi}(0)$ and λ_B

Choice of input parameters

- Shifting only $f_+^{B\pi}(0)$ and λ_B we find:

$$f_+^{B\pi}(0) : 0.28 \pm 0.05 \to 0.22$$
$$\lambda_B : 0.35 \pm 0.15 \to 0.23$$

- these new numbers are very close to what was found using lattice and semi-leptonic CLEO data by Luo and Rosner (2003),

$$|V_{ub}| f_+^{B\pi}(0) = (8.30 \pm 1.6) \times 10^{-4}$$
$$f_+^{B\pi}(0) = 0.23 \pm 0.04$$

- also brings form-factor closer to the SCET value: $0.176 \pm 0.007 \pm ($theory error$)$.

$B \to \pi\pi$ results

The $B \to \pi\pi$ amplitudes are given by ($C = i\,\frac{G_F}{\sqrt{2}} m_B^2 f_\pi f_+^{B\pi}(0)$)

$$\begin{aligned}
\sqrt{2}\,\mathcal{A}_{B^-\to\pi^-\pi^0} &= C\left\{V_{ub}V_{ud}^*\Big[\alpha_1+\alpha_2\Big]\right\},\\
\mathcal{A}_{\bar{B}^0\to\pi^+\pi^-} &= C\left\{V_{ub}V_{ud}^*\Big[\alpha_1+\hat{\alpha}_4^u\Big]+V_{cb}V_{cd}^*\,\hat{\alpha}_4^c\right\},\\
-\,\mathcal{A}_{\bar{B}^0\to\pi^0\pi^0} &= C\left\{V_{ub}V_{ud}^*\Big[\alpha_2-\hat{\alpha}_4^u\Big]-V_{cb}V_{cd}^*\,\hat{\alpha}_4^c\right\},
\end{aligned}$$

the CP-averaged branching fractions ($\times 10^{-6}$)

$$\begin{aligned}
\text{Br}_{B^-\to\pi^-\pi^0} &= 5.5^{+0.3}_{-0.3}(\text{CKM})^{+0.5}_{-0.4}(\text{hadr.})^{+0.9}_{-0.8}(\text{pow.}) && [5.5\pm 0.6],\\
\text{Br}_{\bar{B}^0\to\pi^+\pi^-} &= 5.0^{+0.8}_{-0.9}(\text{CKM})^{+0.3}_{-0.5}(\text{hadr.})^{+1.0}_{-0.5}(\text{pow.}) && [5.0\pm 0.4],\\
\text{Br}_{\bar{B}^0\to\pi^0\pi^0} &= 0.7^{+0.3}_{-0.2}(\text{CKM})^{+0.5}_{-0.2}(\text{hadr.})^{+0.4}_{-0.3}(\text{pow.}) && [1.5\pm 0.3]
\end{aligned}$$

At NLO, QCDF - SCET = ?

What differences remain between the QCD Factorization approach and the SCET approach once NLO corrections are taken into account?

- What do we do with the SCET form factor, $\Xi^{B\to M_1}$? fit (SCET) vs. predict (QCDF)
  - Is perturbation valid at $\sqrt{\Lambda_{\mathrm{QCD}} m_b}$? NLO hard scattering calculations suggest yes.
  - At NLO can no longer keep $\Xi^{B\to M_1}$. Have non-trivial convolution, not simple moments in SCET.
- What do we do with the soft overlap function? fit from $B \to MM$ (SCET) vs use QCD sum rules/lattice (QCDF)
- What do we do with the charming penguins? fit (SCET) vs. assume perturbative & calculate (QCDF)

Conclusions

- Increased precision demands NLO calculations
- NLO in α_s calculations are under way, these will need to be supplemented with full treatment of $1/m_b$ corrections