Hadronic B Decays in PQCD

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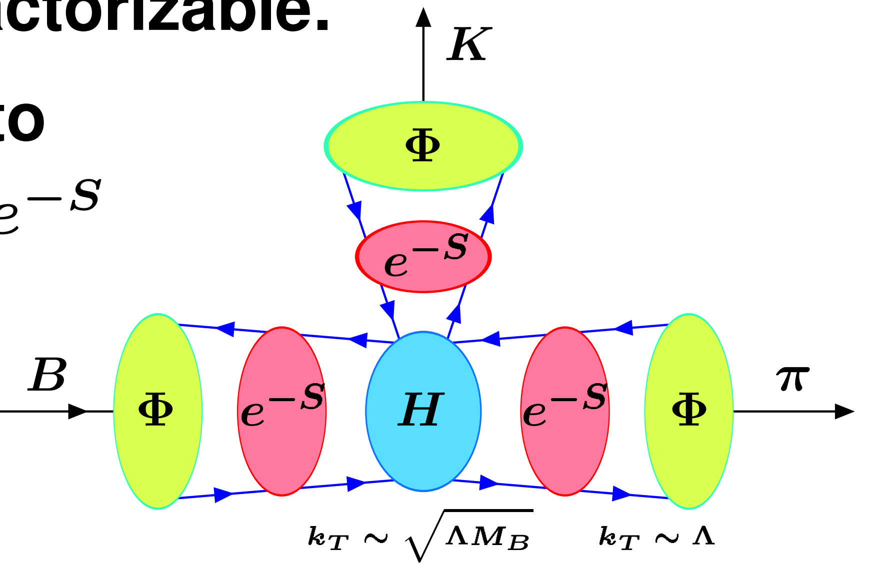
1. Introduction

k_T Factorization for Two-body B Decays

- **Collinear factorization**: QCDF, SCET, $\cdots$
  - End-point singularity ➡ parametrization of amplitudes
- **k_T factorization**: PQCD
  - No end-point singularity due to Sudakov factor *Li,Sterman(92)*

    ➡ All amplitudes are factorizable.
  - Amplitudes can be factorized into a hard part H, Sudakov factors e^{-S} and distribution amplitudes Φ.

    Chang,Li(97); Yeh,Li(97)
  - DAs are inputs from LCSR, Lattice, Data, $\cdots$

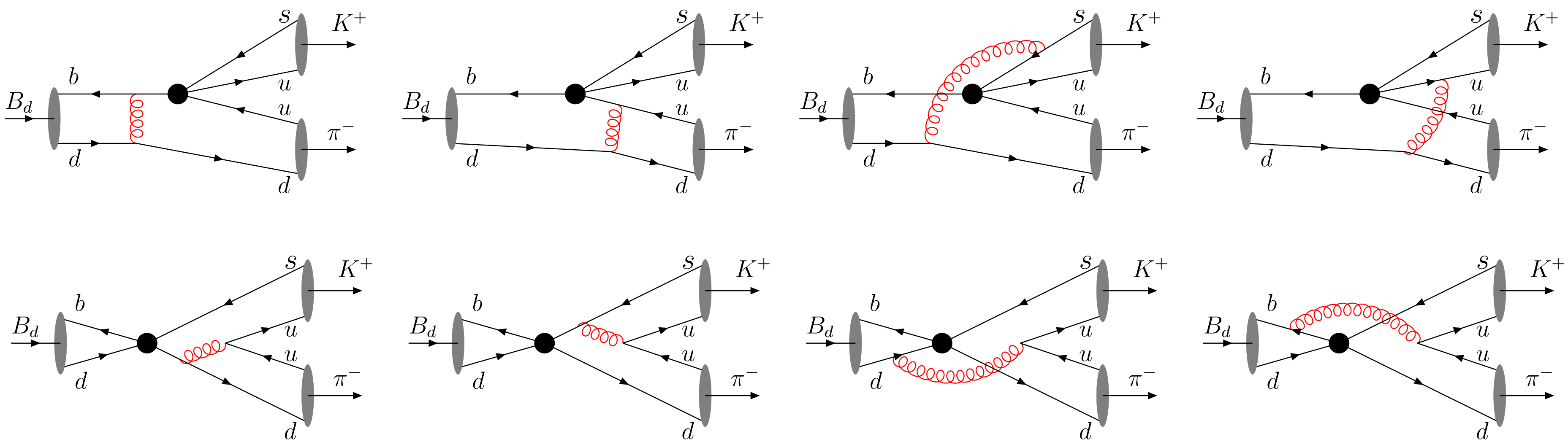


$$\text{Amp.} = \int [dx_i][db_{iT}]\Phi_\pi(x_2)\Phi_K(x_3)\,C(\mu)H(x_i, b_{iT}, \mu)\Phi_B(x_1, b_{1T})\,e^{-S(x_i, b_{iT})}$$

Perturbative QCD Approach

Keum, Li, Sanda (01); Lu, Ukai, Yang (01)

Leading-order diagrams in PQCD



- **The first two diagrams dominate.**
- **Spectator and annihilation diagrams are calculable.**
- **Annihilation diagrams generate a large strong phase.**

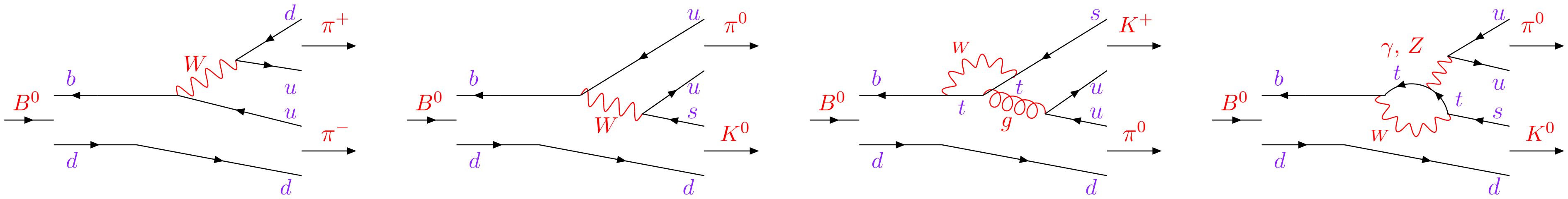
$$A_{CP}(B^0 \to \pi^{\mp} K^{\pm}) = (-21.9 \sim -12.9)\%$$

Data: $A_{CP}(B^0 \to \pi^{\mp} K^{\pm}) = -10.8 \pm 1.7\%$ *HFAG*

2. $B \to \pi K$ and $\pi\pi$ Puzzles

Topological Amplitudes

$T^{(\prime)}$ $\quad C^{(\prime)}$ $\quad P^{(\prime)}$ $\quad P^{(\prime)}_{ew}$



$$
\begin{aligned}
A(B^+ \to \pi^+ K^0) &= P' \\
\sqrt{2}A(B^+ \to \pi^0 K^+) &= -P' - P'_{ew} - (T' + C')\, e^{i\phi_3} \\
A(B^0 \to \pi^- K^+) &= -P' - T' e^{i\phi_3} \\
\sqrt{2}A(B^0 \to \pi^0 K^0) &= P' - P'_{ew} - C' e^{i\phi_3}
\end{aligned}
$$

$$
\begin{aligned}
A(B^0 \to \pi^+ \pi^-) &= -T - P e^{i\phi_2} \\
\sqrt{2}A(B^+ \to \pi^+ \pi^0) &= -T - C - P_{ew} e^{i\phi_2} \\
\sqrt{2}A(B^0 \to \pi^0 \pi^0) &= (P - P_{ew})\, e^{i\phi_2} - C
\end{aligned}
$$

$B \to \pi K$ and $\pi\pi$ Puzzles

- $P' > T'$, $P'_{ew} > C'$ in πK , $T > C$, $P > P_{ew}$ in $\pi\pi$

  ➡ $A_{\rm CP}(\pi^{\mp}K^{\pm}) \simeq A_{\rm CP}(\pi^0 K^{\pm})$

  $\mathrm{Br}(\pi^+\pi^-) \gg \mathrm{Br}(\pi^0\pi^0)$

- **These relations contradict to the data.**

  ➡ **Large $C^{(\prime)}$ and/or large $P_{ew}^{(\prime)}$ are required.**

  Gronau,Rosner(03); Yoshikawa(03); Buras et al.(04); Chiang et al.(04); Ciuchini et al.(04); He,McKellar(04); S.M, Yoshikawa(04); Nandi,Kundu(04); Baek et al.(05); Charng,Li(05); Hou,Nagashima,Soddu(05); Ligeti(05); Wu,Zhou(05); $\cdots$

Br(10^{-6}), $A_{\rm CP}(10^{-2})$	Exp. *HFAG*	QCDF (S4)	LO PQCD
$A_{\rm CP}(B^0 \to \pi^{\mp}K^{\pm})$	-10.8 ± 1.7	-4.1	$-21.9 \sim -12.9$
$A_{\rm CP}(B^{\pm} \to \pi^0 K^{\pm})$	4 ± 4	-3.6	$-17.3 \sim -10.0$
$\mathrm{Br}(B^0 \to \pi^{\mp}\pi^{\pm})$	4.9 ± 0.4	5.2	$5.9 \sim 11.0$
$\mathrm{Br}(B^0 \to \pi^0\pi^0)$	1.45 ± 0.29	0.7	$0.10 \sim 0.65$

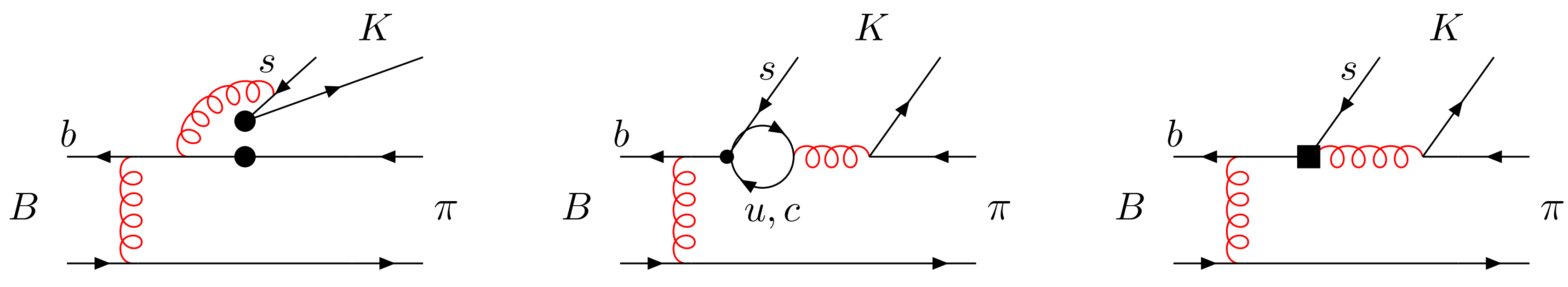
QCDF: Beneke,Neubert(03) LO PQCD: Keum,Li,Sanda(01); Lu,Ukai,Yang(01); Keum,Sanda(03); Keum(04)

Beyond Leading-order Calculations

Li, S.M., Sanda (05)

We include the most important NLO corrections:

- **NLO Wilson coefficients**
- **Vertex Corrections**
- **Quark Loops**
- **Magnetic Penguin**
- **Corrections to form factors are not relevant.**



Enhanced C' in NLO PQCD

Li, S.M., Sanda (05)

- VC enhances and rotates C', which is related to a_2

$$a_2(\mu) = C_1(\mu) + \frac{C_2(\mu)}{N_c} + \frac{\alpha_s(\mu)}{9\pi} V_2(\mu)\, C_2(\mu),$$

and makes $(T' + C')/P'$ almost real.

$$\frac{T'}{P'} \sim 0.19\, e^{-i\,2.8} \Rightarrow 0.15\, e^{-i\,2.8}$$

$$\frac{C'}{P'} \sim 0.01\, e^{i\,0.9} \Rightarrow 0.04\, e^{i\,2.1}$$

$$\frac{P'_{ew}}{P'} \sim 0.16\, e^{i\,0.3} \Rightarrow 0.13\, e^{i\,0.4}$$

$$\frac{T' + C'}{P'} \sim 0.18\, e^{-i\,2.9} \Rightarrow 0.16\, e^{-i\,3.1}$$

$$\Rightarrow A_{CP}(\pi^0 K^\pm) \sim 0$$

$$\left(A_{CP}(\pi^\mp K^\pm) \sim 2\,\mathrm{Im}\left(\frac{T'}{P'}\right) \sin\phi_3 \,, \quad A_{CP}(\pi^0 K^\pm) \sim 2\,\mathrm{Im}\left(\frac{T' + C'}{P'}\right) \sin\phi_3 \right)$$

Resolutions to $B \to \pi K$ Puzzle

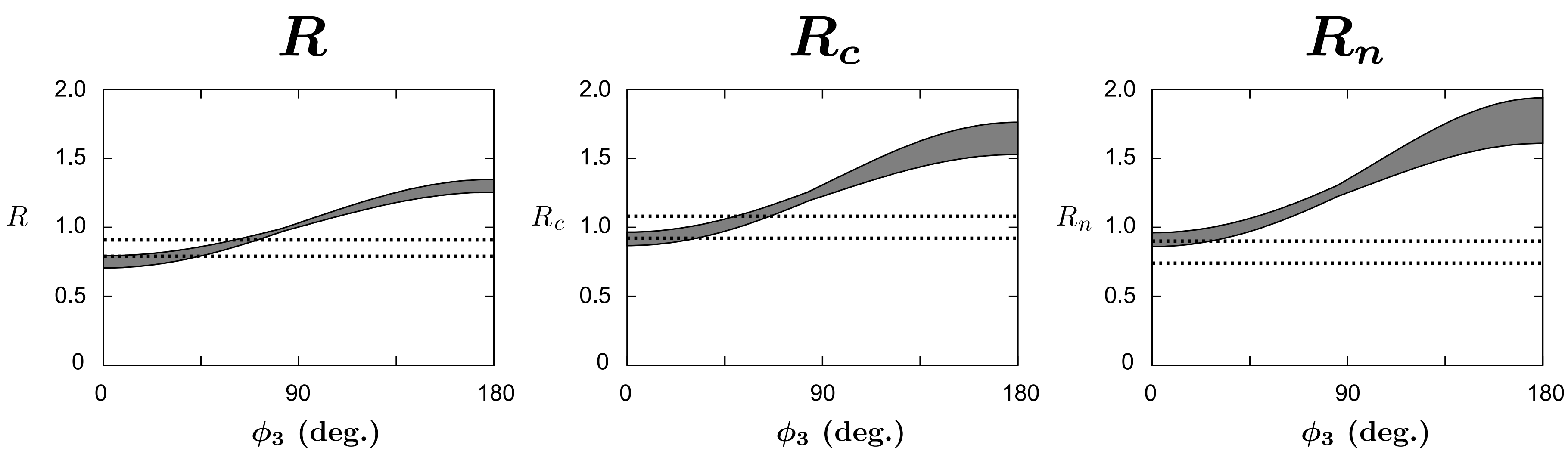
Li, S.M., Sanda (05)

Br(10^{-6}), $A_{\rm CP}(10^{-2})$	Exp. HFAG	LO Keum,Sanda(03)	NLO
$\mathrm{Br}(B^\pm \to \pi^\pm K^0)$	24.1 ± 1.3	$14.4 \sim 26.3$	$23.6^{+14.5}_{-\ 8.4}$
$\mathrm{Br}(B^\pm \to \pi^0 K^\pm)$	12.1 ± 0.8	$7.9 \sim 14.2$	$13.6^{+10.3}_{-\ 5.7}$
$\mathrm{Br}(B^0 \to \pi^\mp K^\pm)$	18.9 ± 0.7	$12.7 \sim 19.3$	$20.4^{+16.1}_{-\ 8.4}$
$\mathrm{Br}(B^0 \to \pi^0 K^0)$	11.5 ± 1.0	$4.5 \sim 8.1$	$8.7^{+\ 6.0}_{-\ 3.4}$
$A_{\rm CP}(B^\pm \to \pi^\pm K^0)$	-2 ± 4	$-1.5 \sim -0.6$	0 ± 0
$A_{\rm CP}(B^\pm \to \pi^0 K^\pm)$	4 ± 4	$-17.3 \sim -10.0$	-1^{+3}_{-6}
$A_{\rm CP}(B^0 \to \pi^\mp K^\pm)$	-10.8 ± 1.7	$-21.9 \sim -12.9$	-10^{+7}_{-8}
$A_{\rm CP}(B^0 \to \pi^0 K^0)$	2 ± 13	$-1.03 \sim -0.90$	-7^{+3}_{-4}

- $A_{CP}(\pi^\mp K^\pm) \not\simeq A_{CP}(\pi^0 K^\pm) \sim 0$
- $B \to \pi K$ **puzzle is resolved.**

Large P'_{ew} ?

Li, S.M., Sanda (05)



$$R = \frac{B(B^0 \to \pi^\mp K^\pm)}{B(B^\pm \to \pi^\pm K^0)} \frac{\tau_{B^+}}{\tau_{B^0}}$$

$$R_c = 2 \frac{B(B^\pm \to \pi^0 K^\pm)}{B(B^\pm \to \pi^\pm K^0)}$$

$$R_n = \frac{1}{2} \frac{B(B^0 \to \pi^\mp K^\pm)}{B(B^0 \to \pi^0 K^0)}$$

- **Smaller R_n still hints a significant P'_{ew}.**

Too small $\mathrm{Br}(B^0 \to \pi^0\pi^0)$

Li, S.M., Sanda (05); Li, S.M. (06)

Br(10^{-6}), $A_{\rm CP}(10^{-2})$	Exp. *HFAG*	LO *Keum,Sanda(03)*	NLO
$\mathrm{Br}(B^0 \to \pi^\mp\pi^\pm)$	4.9 ± 0.4	$5.9 \sim 11.0$	$6.5^{+\,6.7}_{-\,3.8}$
$\mathrm{Br}(B^\pm \to \pi^\pm\pi^0)$	5.5 ± 0.6	$2.7 \sim 4.8$	$4.0^{+\,3.4}_{-\,1.9}$
$\mathrm{Br}(B^0 \to \pi^0\pi^0)$	1.45 ± 0.29	$0.10 \sim 0.65$	$0.29^{+0.50}_{-0.20}$
$A_{\rm CP}(B^0 \to \pi^\mp\pi^\pm)$	37 ± 10	$16.0 \sim 30.0$	18^{+20}_{-12}
$A_{\rm CP}(B^\pm \to \pi^\pm\pi^0)$	1 ± 6	0.0	0 ± 0
$A_{\rm CP}(B^0 \to \pi^0\pi^0)$	28^{+40}_{-39}	$20.0 \sim 40.0$	63^{+35}_{-34}

- **NLO corrections enhance C:** $\frac{C}{T} \sim 0.03\, e^{i\,2.6}$ ➡ $0.19\, e^{-i\,1.1}$
- **Not enough to resolve $B \to \pi\pi$ puzzle.** ($|C| \approx |T|$)

$B \to \pi\pi$ Puzzle

Li, S.M. (06)

- Br$(B^0 \to \rho^0\rho^0)$ is also sensitive to C.

Br(10^{-6})	BABAR	Belle	LO	NLO
Br$(B^0 \to \rho^\mp\rho^\pm)$	$30 \pm 4 \pm 5$	$22.8 \pm 3.8^{+2.3}_{-2.6}$	27.8	$25.3^{+25.3}_{-13.8}$
Br$(B^\pm \to \rho^\pm\rho^0)$	$17.2 \pm 2.5 \pm 2.8$	$31.7 \pm 7.1^{+3.8}_{-6.7}$	13.7	$16.0^{+15.0}_{-\ 8.1}$
Br$(B^0 \to \rho^0\rho^0)$	< 1.1	—	0.33	$0.92^{+1.10}_{-0.56}$

- Br$(B^0 \to \rho^0\rho^0)$ is saturated by NLO corrections.
- The $B \to \pi\pi$ puzzle is confirmed.
- All proposed resolutions should survive the constraints from other data.

Comparisons with Other Approaches

- **QCDF**
  - $T' + C'$ is not parallel to P'.
    - ➡ $A_{CP}(B^{\pm} \to \pi^0 K^{\pm}) \not\approx 0$ *Beneke,Neubert(03)*
  - C is enhanced by the NLO jet function in SCET. *Beneke,Yang(05)*
    - ➡ Large Br($B^0 \to \pi^0\pi^0$) but overshoot Br($B^0 \to \rho^0\rho^0$). *Li, S.M.(06)*
- **SCET** *Bauer,Pirjol,Rothstein,Stewart(04); Bauer,Rothstein,Stewart(05); Williamson,Zupan(06)*
  - $C^{(\prime)}/T^{(\prime)}$ is real at LO.
    - ➡ $(T' + C')/P'$ is parallel to T'/P'.
    - ➡ $-A_{CP}(B^0 \to \pi^{\mp} K^{\pm}) \lesssim -A_{CP}(B^{\pm} \to \pi^0 K^{\pm})$
  - $|C^{(\prime)}/T^{(\prime)}| \approx 1$
    - ➡ Large Br($B^0 \to \pi^0\pi^0$).

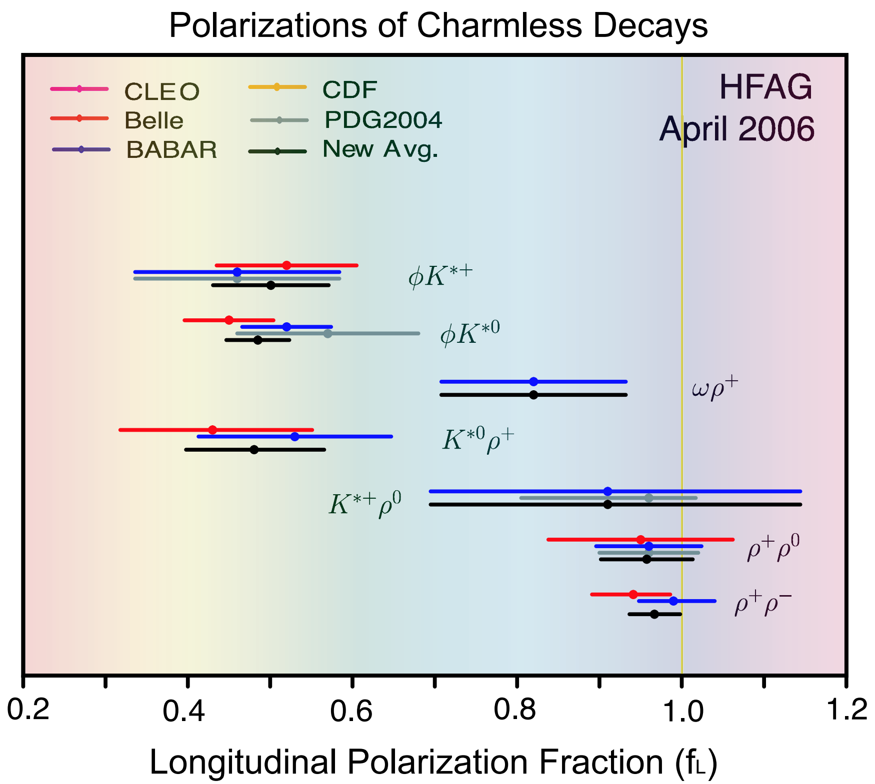
      Br($B^0 \to \rho^0\rho^0$) should be checked.

3. Polarizations in $B \to VV$

Small Transverse Polarizations in $B \to \phi K^*$

- **Polarization fractions from naïve factorization**

$$R_L \sim 1 - \mathcal{O}(m_V^2/m_B^2) \quad \gg \quad R_{\parallel} \sim R_{\perp} \sim \mathcal{O}(m_V^2/m_B^2)$$



$$R_{L,\parallel,\perp} = \frac{|\mathcal{A}_{L,\parallel,\perp}|^2}{|\mathcal{A}_L|^2 + |\mathcal{A}_\parallel|^2 + |\mathcal{A}_\perp|^2}$$

➡ **Anomaly in ϕK^* and $K^{*0}\rho^+$.**

Polarizations in LO PQCD

Chen, Keum, Li (02); Li, S.M. (04); Huang et al. (05)

- **Penguin-dominated modes**
  - ➡ **Subleading contributions are important.**

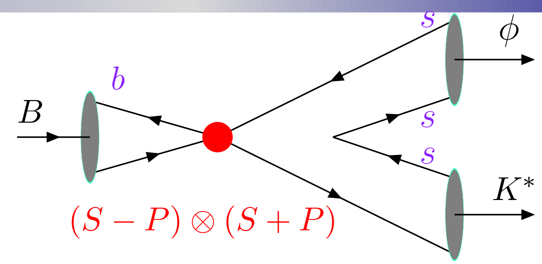
    Annihilation from $(S-P)(S+P)$: $R_L \sim R_{||} \sim R_\perp$

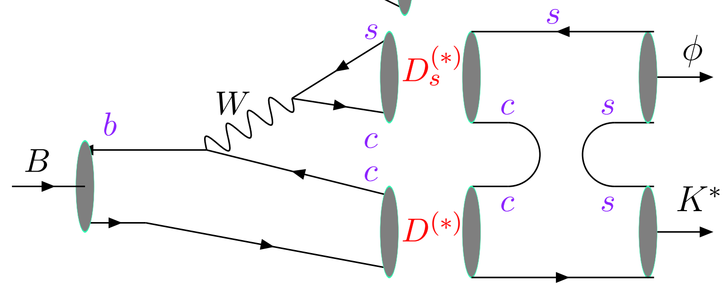
		R_L	$R_{\|\|}$	$R_\perp$	$\phi_{\|\|}$ (rad)	$\phi_\perp$ (rad)
ϕK^{*0}	Leading	0.923	0.040	0.035	π	π
	With Subleadings	0.750	0.135	0.115	2.55	2.54
ϕK^{*+}	Leading	0.923	0.040	0.035	π	π
	With Subleadings	0.748	0.133	0.111	2.55	2.54

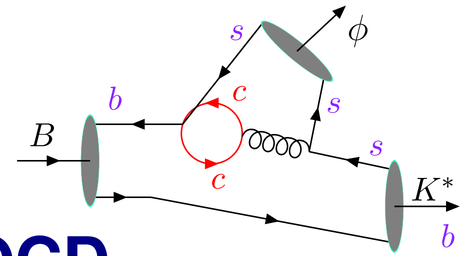
- $K^{*0}\rho^+$ = **Pure-penguin** ➡ **almost same as ϕK^***
- $K^{*+}\rho^0$ ➡ $R_L \sim 0.85$ **due to Tree** ➡ **agree with data**

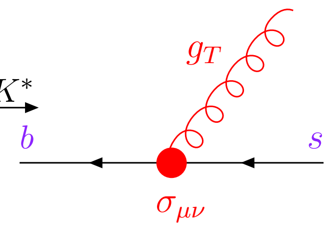
  ➡ **Cannot explain ϕK^* and $K^{*0}\rho^+$**

Proposals for Explanation of $B \to \phi K^*$ Anomaly









- **Penguin Annihilation** *Kagan(04)*
- **Rescattering effect**
  Colangelo,Fazio,Pham(04);
  Ladisa,Laporta,Nardulli,Santorelli(04);
  Cheng,Chua,Soni(05)
- **Charming Penguin**
  Bauer,Pirjol,Rothstein,Stewart(04)
- **$b \to sg$ Dipole Penguin**
  Hou,Nagashima(04)
- **Small $B \to K^*$ form factor in PQCD** *Li(04)*
  - **Smaller $A_0^{B \to K^*}$ ➡ $R_L \sim 0.6$ for ϕK^{*0}**
- **Modified definition of hard scale in PQCD** *Chen(06)*
  - **R_L for ϕK^* could approach to 0.6. But $R_L \sim 0.8$ for $K^{*0}\rho^+$.**
- **or New Physics ?**

4. Mixing-induced CP asymmetries

Possible SM Deviation for $S_{\pi^0 K_S}$

$$A_{CP}(B^0(t) \to \pi^0 K_S) \equiv \frac{B(\bar{B}^0(t) \to \pi^0 K_S) - B(B^0(t) \to \pi^0 K_S)}{B(\bar{B}^0(t) \to \pi^0 K_S) + B(B^0(t) \to \pi^0 K_S)}$$

$$= A_{\pi^0 K_S} \cos(\Delta M_d\, t) + S_{\pi^0 K_S} \sin(\Delta M_d\, t) ,$$

$$A_{\pi^0 K_S} = \frac{|\lambda_{\pi^0 K_S}|^2 - 1}{1 + |\lambda_{\pi^0 K_S}|^2} , \qquad S_{\pi^0 K_S} = \frac{2\,\mathrm{Im}(\lambda_{\pi^0 \mathrm{K_S}})}{1 + |\lambda_{\pi^0 \mathrm{K_S}}|^2}$$

$$\lambda_{\pi^0 K_S} = -e^{-2i\phi_1} \frac{P' - P'_{ew} - C' e^{-i\phi_3}}{P' - P'_{ew} - C' e^{i\phi_3}}$$

- **If C' is sizable, $S_{\pi^0 K_S} \neq S_{c\bar{c}s}$.**
- **Possible SM deviation in LO PQCD and NLO PQCD**

$$S_{\pi^0 K_S} - S_{c\bar{c}s} = 0.02 \;\Rightarrow\; 0.06^{+0.02}_{-0.03} > 0$$ *Li,S.M(05)*

Data: $S_{\pi^0 K_S} = 0.31 \pm 0.26$ *HFAG*

5. Summary

- NLO corrections enhance $C^{(\prime)}$.
  - The puzzle in $A_{CP}(\pi K)$ was resolved.
  - The puzzle of Br$(\pi^0\pi^0)$ still stands.
- Small R_L for ϕK^* might come from QCD uncertainties.
- The possible SM deviation of $S_{\pi^0 K_S}$ is

$$S_{\pi^0 K_S} - S_{c\bar{c}s} = 0.06^{+0.02}_{-0.03} \; > \; 0$$

- It is important to study NLO corrections to other modes.

  Polarizations for ϕK^* and $K^*\rho$, $S_{\phi K_S}$, $\cdots$

Backup

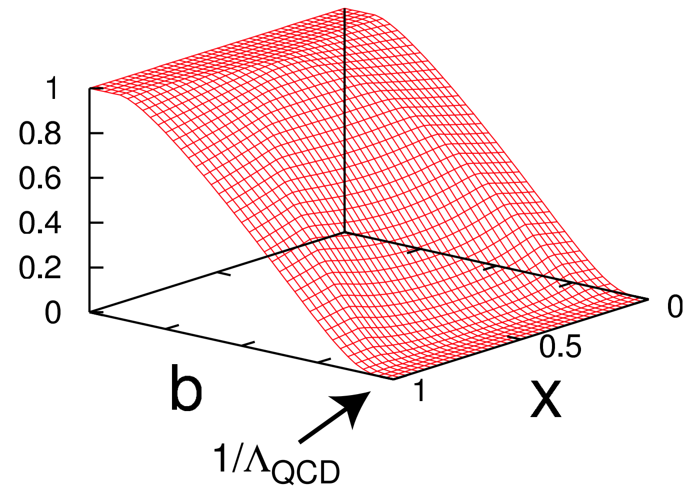
Sudakov Factor in QCD

Botts, Sterman (89); Li, Sterman (92)

When we retain k_T, the large double logarithms are generated from the overlap of collinear and soft divergence in radiative corrections to mesons.

- **Sudakov factor**
- Large b (small k_T) is suppressed.

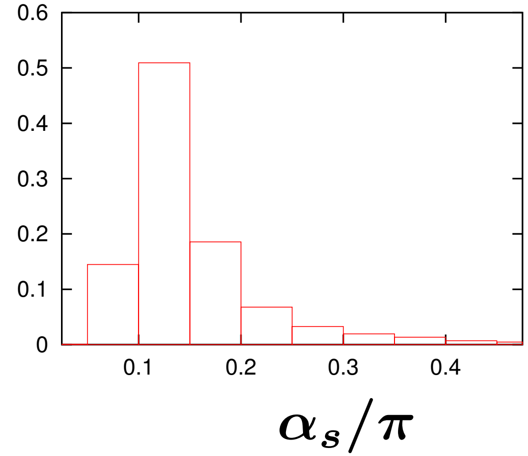
$$\Psi(P, \mathbf{k}_T) = \int d^2\mathbf{b}\, e^{i\,\mathbf{k}_T\cdot\mathbf{b}}\, \Psi(P, \mathbf{b})$$



- SF ensures a perturbative calculation of hard part.

$$\frac{1}{q^2} = \frac{1}{-x_1 x_2 M_B^2 - |\mathbf{k}_{1T} - \mathbf{k}_{2T}|^2}$$

No End-Point Singularity !!



Physical Meaning of Sudakov Factor in *b* space

- **If a single quark interacts with a hard gluon, it must emit many collinear gluons.**

  ➡ **Not exclusive processes !!**

- **If a $q\bar{q}$ pair with small separation b interacts with a hard gluon, it emits no gluons since it is a color singlet.**
- **The Sudakov factor gives a probability for no emitted gluons.**