Theory of $\bar{B} \to X_u \, l^- \, \bar{\nu}$ decays and $|V_{ub}|$

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Outline

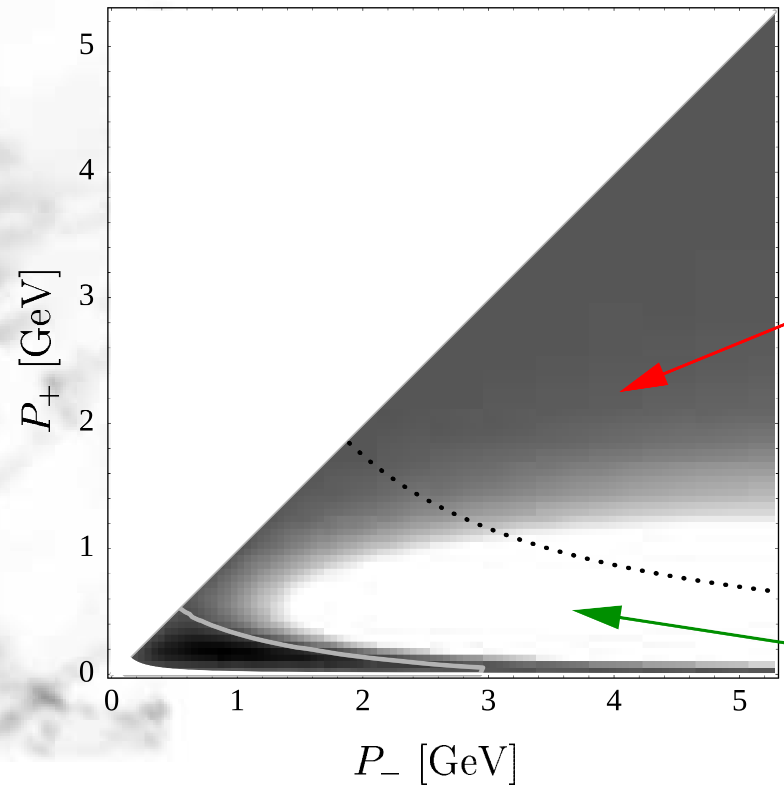
1. Direct calculation of partial decay rates in the “shape-function region”.
2. Relations between $\bar{B} \to X_u\, l^- \bar{\nu}$ partial rates and $\bar{B} \to X_s \gamma$ photon spectrum.

Motivation

Extraction of $|V_{ub}|$, UT side opposite β.

Kinematics of $\bar{B} \to X_u\, l^-\, \bar{\nu}$

Three independent variables, e.g. energy E_l of the charged lepton, and light-cone momenta $P_\pm = E_X \mp |\vec{P}_X|$ of the final hadronic state.



Large charm background

$P_+P_- > M_D^2$

No charm background

$P_+P_- < M_D^2$

- Accurate measurements only for charmfree region.
- Typically where $P_+ \sim \mathcal{O}(\Lambda_{\rm QCD})$,
  and $P_- \sim \mathcal{O}(m_b)$.
- Separation of physics at different scales:

$$\mu_h \sim m_b, \quad \mu_i \sim \sqrt{m_b \Lambda_{\rm QCD}}, \quad \Lambda_{\rm QCD}$$

- Based on universal **QCD-factorization** formula

$$d\Gamma = H\,J \otimes \hat{S} + \frac{1}{m_b} H_i' J_i' \otimes \hat{S}_i' + \ldots$$

Korchemsky, Sterman; Phys. Lett. B**340**, 96 (1994)

Lee, Stewart; Nucl. Phys. B **721**, 325 (2005)

Bosch, Neubert, Paz; JHEP **0411**, 073 (2004)

The fully differential decay rate can be written as

$$\frac{d^3\Gamma_u}{dP_+\,dP_-\,dP_l} = \frac{G_F^2|V_{ub}|^2}{16\pi^3}\,U_y(\mu_h,\mu_i)\,(M_B-P_+)\Big[(P_- - P_l)(M_B - P_- + P_l - P_+)\,\mathcal{F}_1 + (M_B-P_-)(P_- - P_+)\,\mathcal{F}_2 + (P_- - P_l)(P_l - P_+)\,\mathcal{F}_3\Big]$$

without explicit reference to partonic quantities. Here, $y = (P_- - P_+)/(M_B - P_+)$

$\mathcal{F}_i$ are factorized in a QCD $\longrightarrow$ SCET $\longrightarrow$ HQET matching procedure:

- Leading Power: [Bauer, Manohar; Phys. Rev. D **70**, 034024 (2004); Bosch, B.O.L., Neubert, Paz; Nucl. Phys. B **699**, 335 (2004)]

$$\mathcal{F}_i^{(0)}(P_+,y) = H_{ui}(y,\mu_h)\int_0^{P_+} d\hat\omega\, y m_b\, J(y m_b(P_+-\hat\omega),\mu_i)\,\hat S(\hat\omega,\mu_i)$$

- Subleading Power corrections from subleading shape functions (hadronic corrections) and kinematical corrections.

Why do this?

- Systematic separation of effects at different scales.
- EFTs have more symmetry $\Rightarrow$ $\hat{S}(\hat{\omega})$ unique!

$\Rightarrow$ For example, $\bar{B} \to X_s\gamma$ spectrum factorizes similarly. [Neubert, Eur. Phys. J. C **40**, 165 (2005)]

- Resummation of large logs.

$\alpha_s^n \ln^k \frac{\mu_h}{\mu_i}, \qquad k = 2n, 2n-1, \ldots$ ("Sudakov factor")

1-loop matching, 2-loop running, 3-loop cusp anomalous dimension

Experimentalists like to have predictions for entire phase-space, so we need . . .

Connection with "OPE region"

If $$\int_0^{\Delta} d\hat{\omega}\; f(\hat{\omega})\hat{S}(\hat{\omega})\;, \quad \Delta \gg \Lambda_{\rm QCD}$$

- can perform OPE in $\Lambda_{\rm QCD}/\bar{\Delta}$, $\alpha_s(\bar{\Delta})$, where $\bar{\Delta} = \Delta - \bar{\Lambda}$.

  Leads to "Multi Scale OPE" [Neubert, Phys. Lett. B **612**, 13 (2005)]

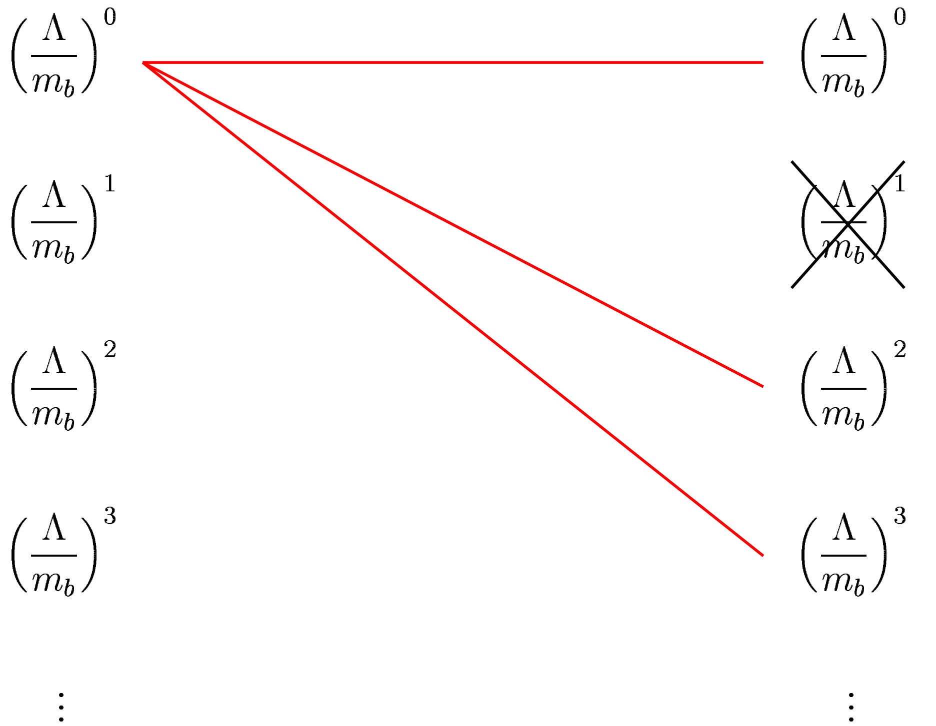
- sensitive to moments of $\hat{S}$.

But: different power counting, **nontrivial** transition.

Anatomy of partial rate $\int_0^{\Delta} dP_+ \frac{d\Gamma}{dP_+}$

$\Delta \sim \Lambda_{\mathrm{QCD}}$ (SF region) , $\Delta \gg \Lambda_{\mathrm{QCD}}$ (OPE region)

When integrating the **tree-level**, leading-power factorized expression over a large phase space $\longrightarrow$ tree-level, leading-power OPE result. Also feeds into subleading power on the OPE side. (Via moments of the leading shape function.)



Anatomy of partial rate $\int_0^{\Delta} dP_+ \frac{d\Gamma}{dP_+}$

This pattern repeats itself at each higher order in power counting.

(E.g. first subleading shape functions have zero norm, but nonzero first moment.)

[Bosch, Neubert and Paz; JHEP **0411**, 073 (2004)]

[Lee, Stewart; Nucl. Phys. B **721**, 325 (2005)]

[Beneke, Campanario, Mannel, Pecjak; JHEP **0506**, 071 (2005)]

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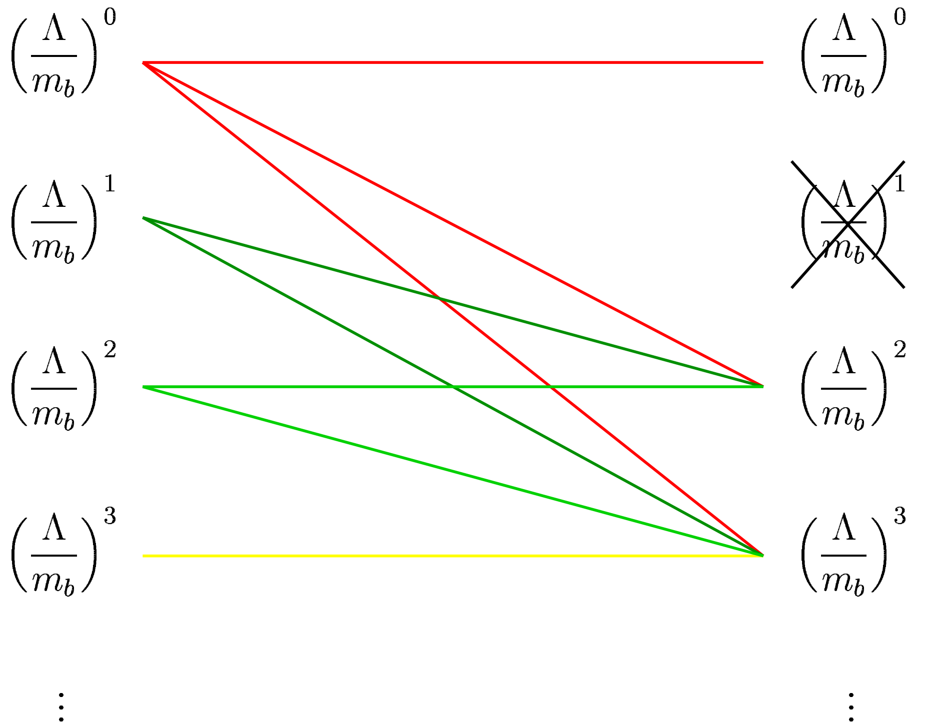

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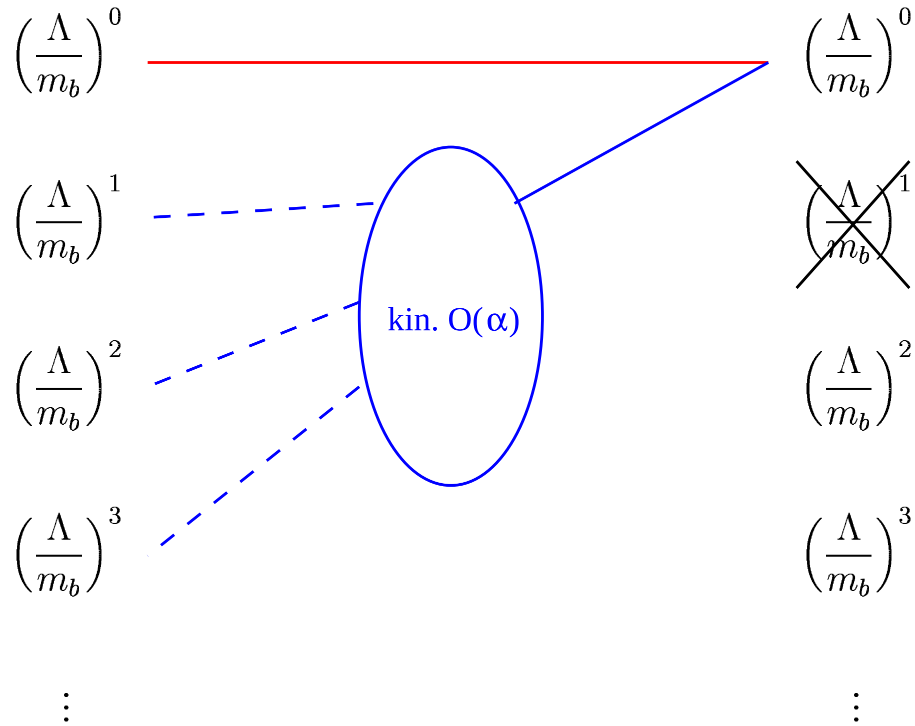
Including **radiative corrections** to the factorized leading-power decay rate and increasing Δ does not reproduce the OPE result. Starting at $\mathcal{O}(\alpha_s)$ some terms are missing because they are suppressed for small Δ. These are “kinematical corrections”.

[De Fazio, Neubert; JHEP **9906**, 017 (1999)]



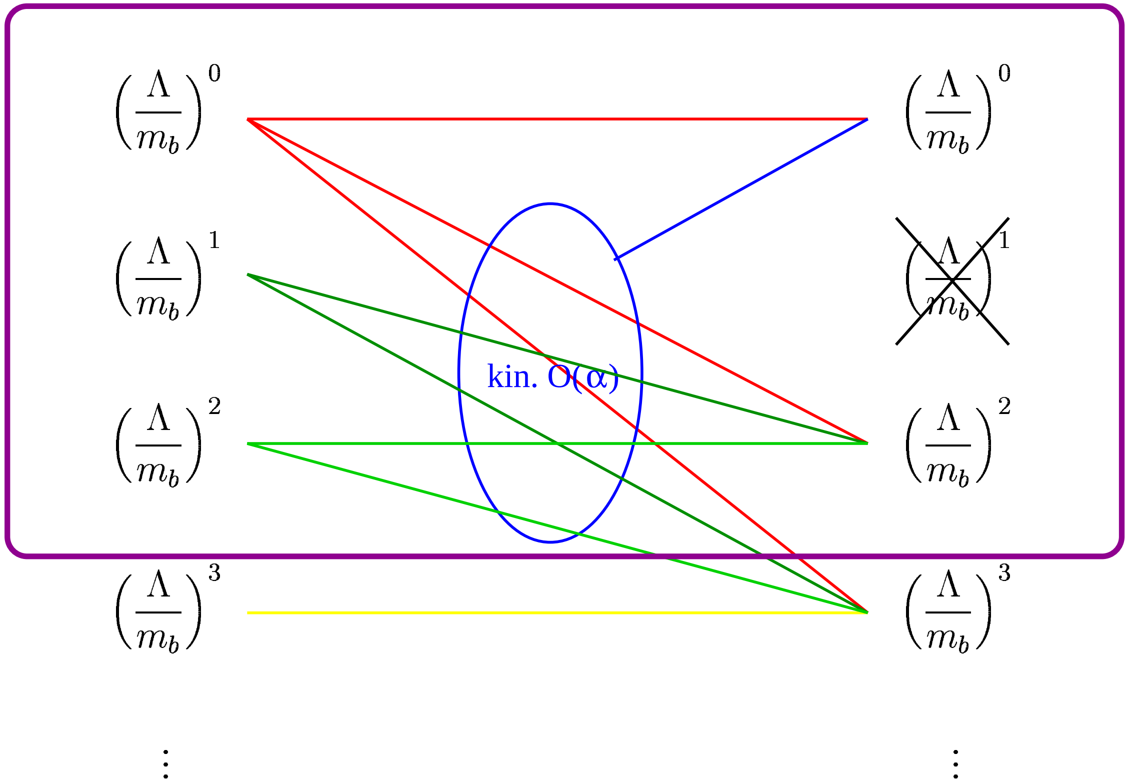
Anatomy of partial rate $\int_0^\Delta dP_+ \frac{d\Gamma}{dP_+}$

$\Delta \sim \Lambda_{\rm QCD}$ (SF region) , $\Delta \gg \Lambda_{\rm QCD}$ (OPE region)

By including all contributions in the purple box the difference between the factorized expressions (applied for large Δ) and a standard OPE is analytically of order $(1/m_b)^3$ and numerically negligible.

This has been incorporated in

[B.O.L., Neubert, Paz: Phys. Rev. D **72**, 073006 (2005)]



Modelling the shape functions

Predictions require functional forms for the shape functions.

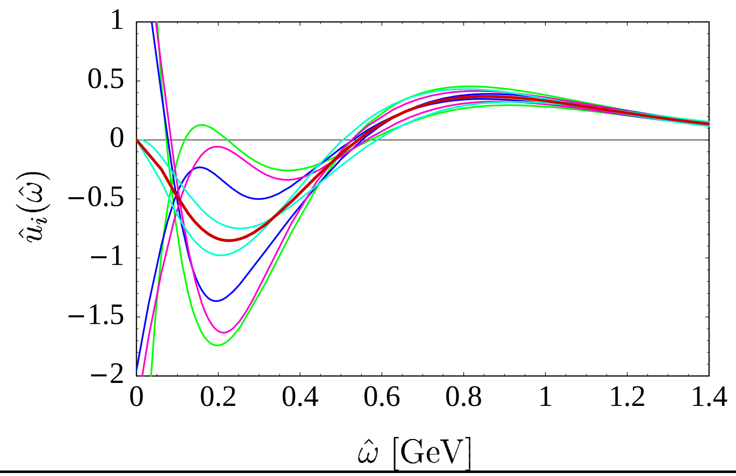
- Leading shape function extracted from $\bar{B} \to X_s \gamma$ photon spectrum.

For this absorb subleading shape functions into $\hat{S}$.

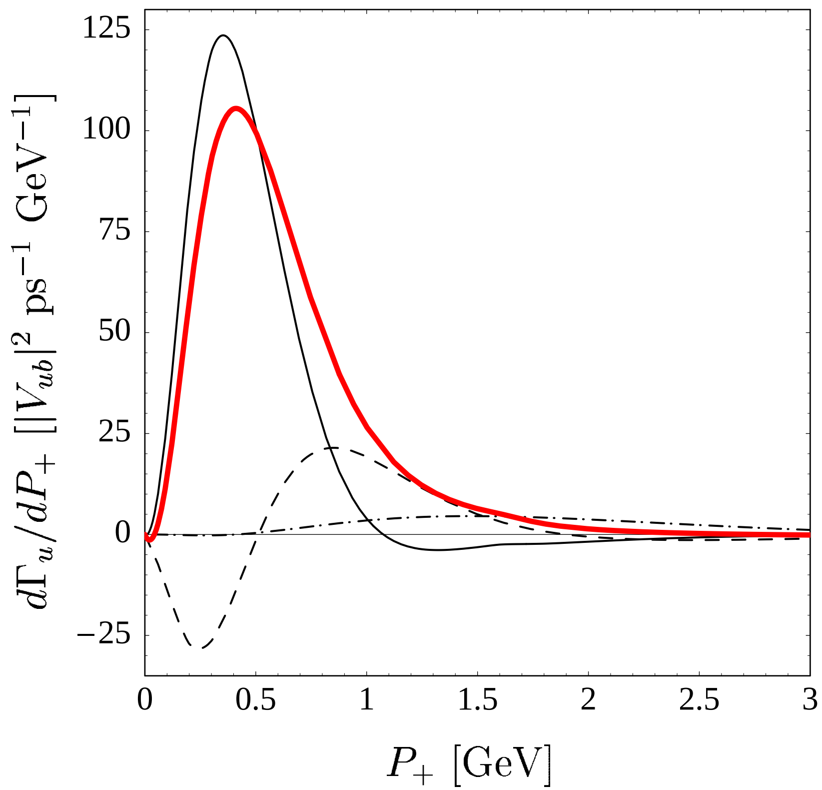
Can $\bar{B} \to X_c l^- \bar{\nu}$ spectra help? [Boos, Feldmann, Mannel, Pecjak; hep-ph/0512157]

(In particular $U = P_+ - m_c^2/p_-$ spectrum ?)

- Only available information for subleading shape functions are their first few moments. So we model and estimate uncertainty.



Example: P_+ spectrum



The P_+ spectrum extended to large values of P_+. The thin solid line denotes the leading-power prediction, the dashed line depicts first-order power corrections, the dash-dotted line shows second-order power corrections, and the thick solid line is their sum.

Example: Cut on $P_+ \leq 0.65$ GeV

- The central value assumes perfect extraction of $\hat{S}(\hat{\omega})$.

$$\Gamma_u(0.65\ \mathrm{GeV}) = (45.3 \pm 2.5\,\text{[pert]} \pm 1.5\,\text{[hadr]} \pm 1.3\,\text{[WA]})|V_{ub}|^2\,\mathrm{ps}^{-1}$$

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  Can be tested with cut on *high* q^2. [B.O.L., Neubert, Paz: Phys. Rev. D **72**, 073006 (2005)]

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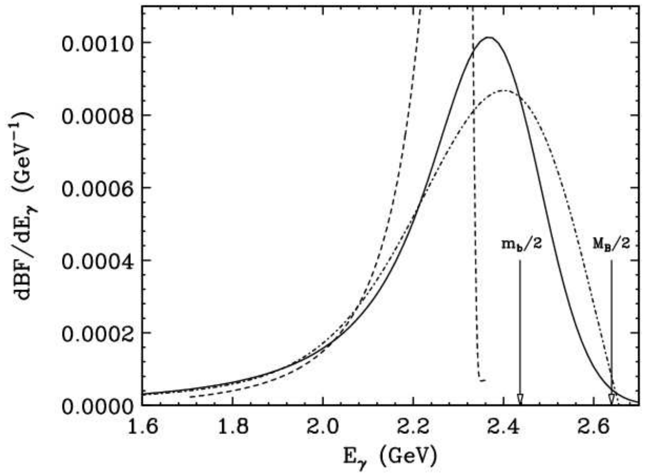
Briefly: a different approach

Recently a calculation called "Dressed Gluon Exponentiation" appeared.

[Andersen, Gardi; JHEP **0506** 030 (2005); JHEP **0601** 097 (2006)]



- Instead of B-meson state, DGE uses on-shell b-quark "dressed" with gluons.
- Nevertheless, kinematic range extends beyond partonic phase space.
- Assumes models for exact anomalous dimensions of jet and soft functions, motivated by large β_0 limit and cancellation of certain renormalon ambiguities.
- Claims that then there is no need for non-perturbative functions until very endpoint of spectrum.

This is a model calculation.

Shape-function free relations

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Extracting the leading shape function from $\bar{B} \to X_s \gamma$ data

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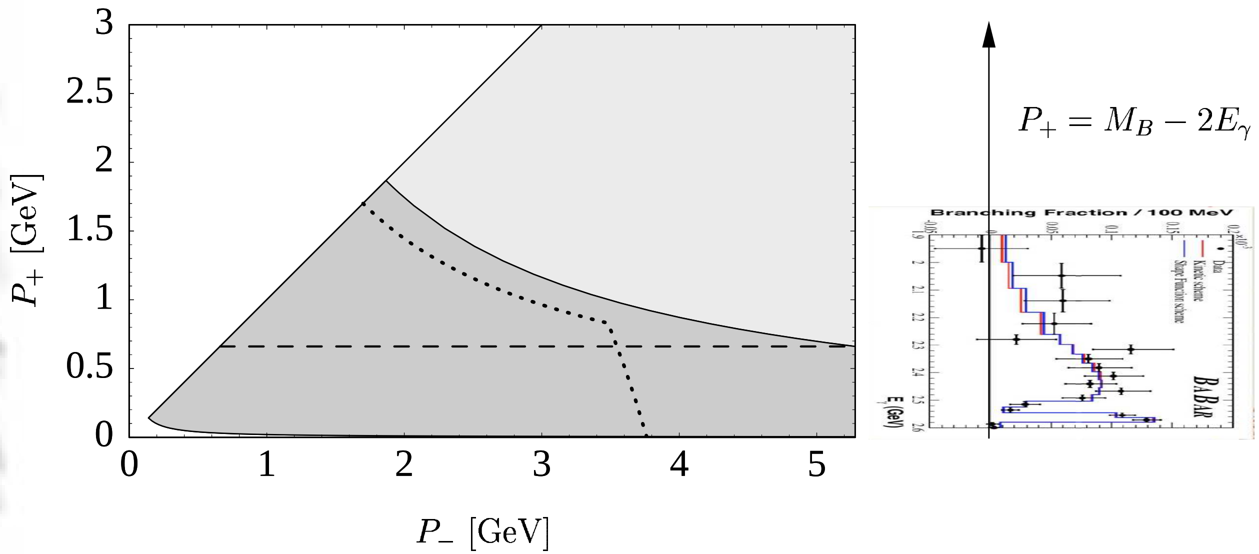
Complementary way to the direct calculation of partial decay rates.

We can make full use of QCD factorization theorems for the spectra!

Elimination of the shape function.

$\bar{B} \to X_u \, l^- \bar{\nu}$ $\bar{B} \to X_s \, \gamma$



$$\underbrace{\Gamma_u \Big|_{\text{cut}}}_{\text{Exp. Input}} = |V_{ub}|^2 \int_0^{\Delta} dP_+ \, W(\Delta, P_+) \; \underbrace{\frac{1}{\Gamma_s} \frac{d\Gamma_s}{dP_+}}_{\text{Exp. Input}} \; + \; \text{pow. corr.}$$

Inclusive semileptonic B decays

- The idea was put forward in [Neubert, Phys. Rev. D **49**, 4623 (1994)],
- further work done for cuts on lepton energy

  [Leibovich, Low, Rothstein, Phys. Rev. D **61**, 053006 (2000)],
- hadronic invariant mass [Leibovich, Low, Rothstein, Phys. Lett. B **486**, 86 (2000)],
- hadronic P_+ [Hoang, Ligeti, Luke, Phys. Rev. D **71**, 093007 (2005)],

  [B.O.L., Neubert, Paz, JHEP **0510**, 084 (2005)],
- *any* [B.O.L., JHEP **0601**, 104 (2006)].

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- Using QCD factorization theorems
  $d\Gamma^{(0)} \sim H(\mu_h) J(\mu_i) \otimes \hat{S}(\mu_i)$ we can show that
  $W^{(0)} \sim H(\mu_h) \otimes Y(\mu_i)$,
  where the kernel Y is derived from the jet function J.

  [**To complete 2-loop order!**]

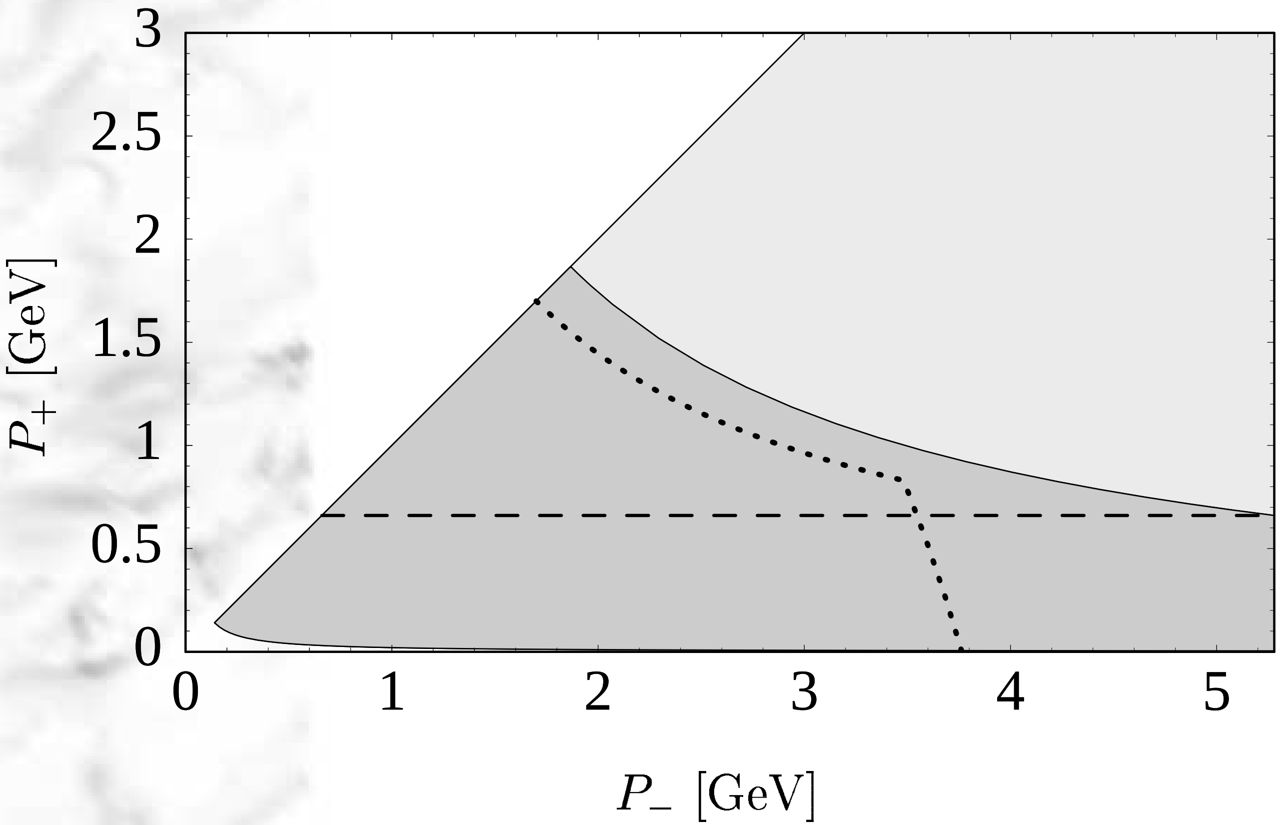
- The cut is encoded in the convolution between H and Y.

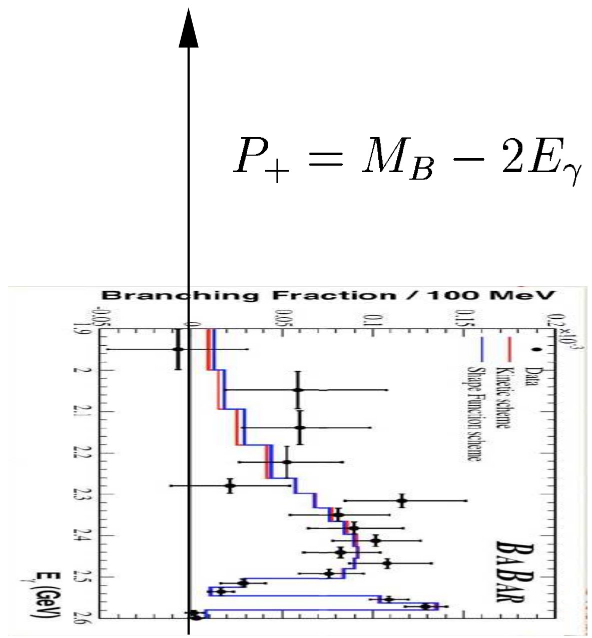
  [**Automation!**]

For Reference:

$\bar{B} \to X_u \, l^- \bar{\nu}$

$\bar{B} \to X_s \, \gamma$





Some Examples

[B.O.L., hep-ph/0511098]

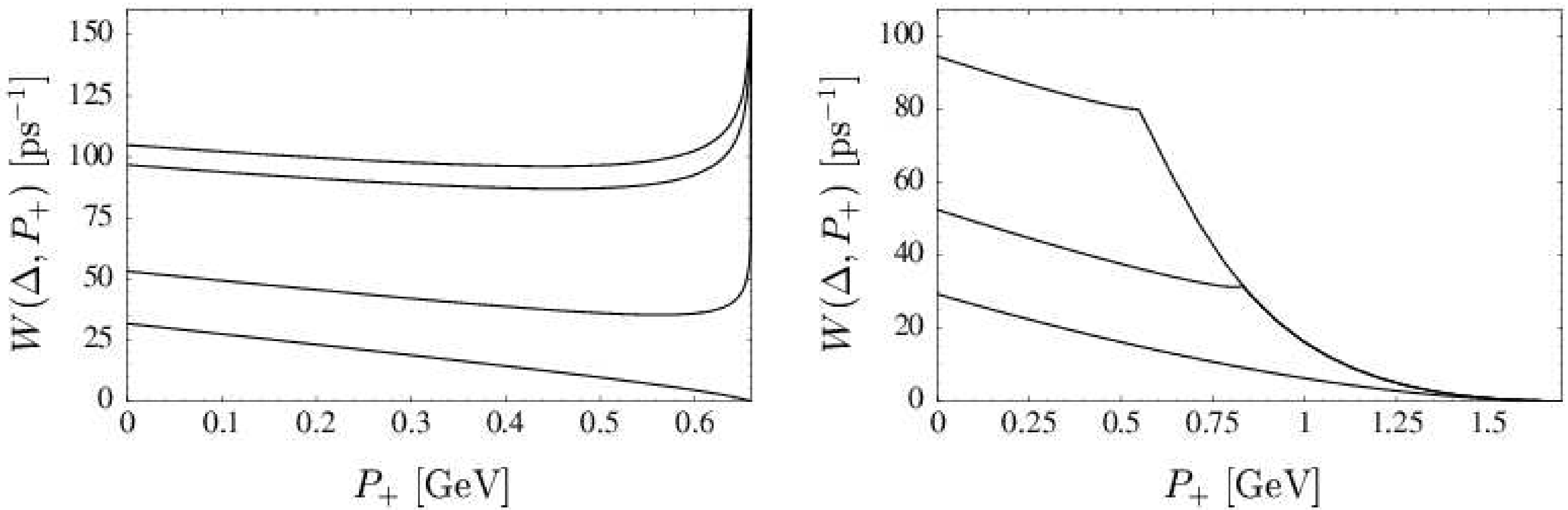


Figure 2: Examples of the weight function for different kinematic cuts. LEFT: Cutting on $P_+ \leq \Delta = 0.66$ GeV and $E_l > E_0$. From top to bottom the four functions are for $E_0 = 0$, $E_0 = 1$ GeV, $E_0 = 2$ GeV, and $E_0 = (M_B - \Delta)/2$. RIGHT: Cutting on $M_X \leq M_0 = 1.7$ GeV, $q^2 > q_0^2$, and $E_l > 1$ GeV. The three functions are for $q_0^2 = 0$ (top), $q_0^2 = 8$ GeV^2 (middle), and $q_0^2 = (M_B - M_0)^2$ (bottom).

For cuts on hadronic mass need photon spectrum over wide range

Fortunately, the weight function is small there.

Cutting away further events in the low-P_+ region makes no sense here.

“Pure” cut on P_+ most promissing.

Cutting soft leptons away doesn’t hurt much.

We are looking forward to the fi rst implementation by BaBar and Belle.

Example: Cut on $P_+ \leq 0.65$ GeV

- Perturbative contribution and uncertainty from NNLO @ μ_i as important as NLO @ μ_h.

$$\begin{aligned}\Gamma_u(0.65\,\mathrm{GeV}) &= (46.5 \pm 1.4_{\,[\mathrm{pert}]} \pm 1.8_{\,[\mathrm{hadr}]} \pm 1.8_{\,[m_b]} \pm 0.8_{\,[\mathrm{pars}]} \pm 2.8_{\,[\mathrm{norm}]})\,|V_{ub}|^2\,\mathrm{ps}^{-1} \\ &= (46.5 \pm 4.1)\,|V_{ub}|^2\,\mathrm{ps}^{-1}, \qquad (45)\end{aligned}$$

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Overall theoretical error on $|V_{ub}|$ thus far: 4-5%.

Summary:

$$d\Gamma = HJ \otimes \hat{S} + \frac{1}{m_b} H_i' J_i' \otimes \hat{S}_i' + \ldots$$

$$W = H_\Gamma H_u \otimes Y + \ldots$$

End of Summary

What could be next?

[Just some thoughts, no promisses implied!]

- Perturbative corrections:
  - J at 2-loop (most important). Y already at 2-loop.
  - H at 2-loop: matching $V - A$ current to SCET current. For $\bar{B} \to X_s \gamma$ at NNLO major effort.
  - Resummation (Sudakov) improvement to next level requires 3- and 4-loop running.
  - Perturbative corrections at subleading power.
- Shape functions:
  - More terms in OPE of shape-function moments? (Mainly for MSOPE)
  - More terms in OPE of subleading shape-function moments?

Thank you.