Lattice QCD Progress and Outlook

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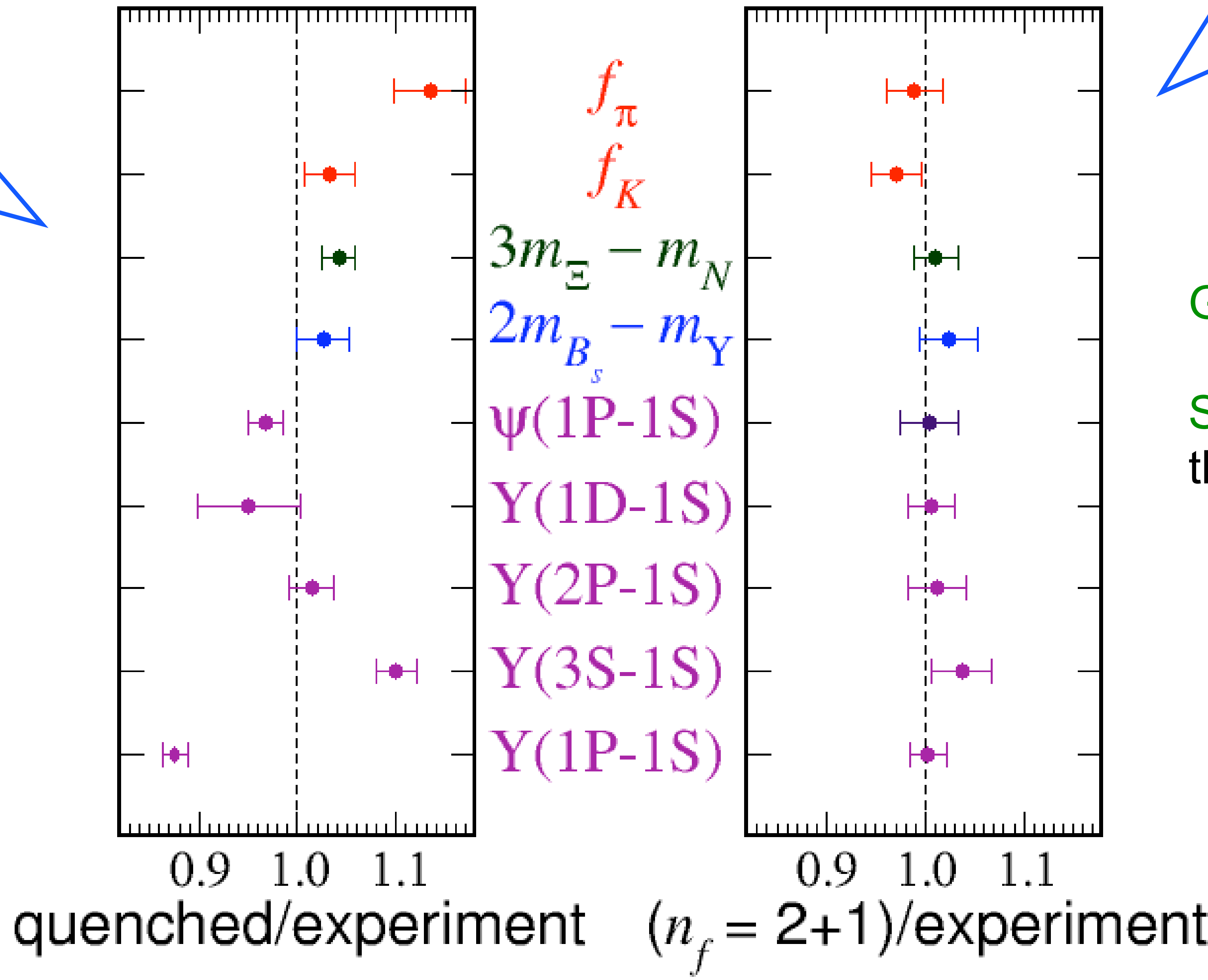
Lattice QCD calculations have made terrific progress in recent years.

- Simple quantities agree with experiment to a few %.
- A few quantities have been predicted ahead of experiment.
- Lattice calculations are playing an increasingly essential role in analysis of experiment.

Quantities that used to agree decently, ~10%, in the quenched approximation...

... agree to a few % in recent unquenched calculations.



Gold-plated quantities.

Staggered fermions, the least CPU-intensive.

“Gold-plated quantities” of lattice QCD

Quantities that are easiest for theory and experiment to both get right.

Stable particle, one-hadron processes. Especially mesons.

More complicated methods are required for multihadron processes:
- unstable particles are messy to interpret,
- multihadron final states are different in Euclidean and Minkowski space.

Three families of lattice fermions

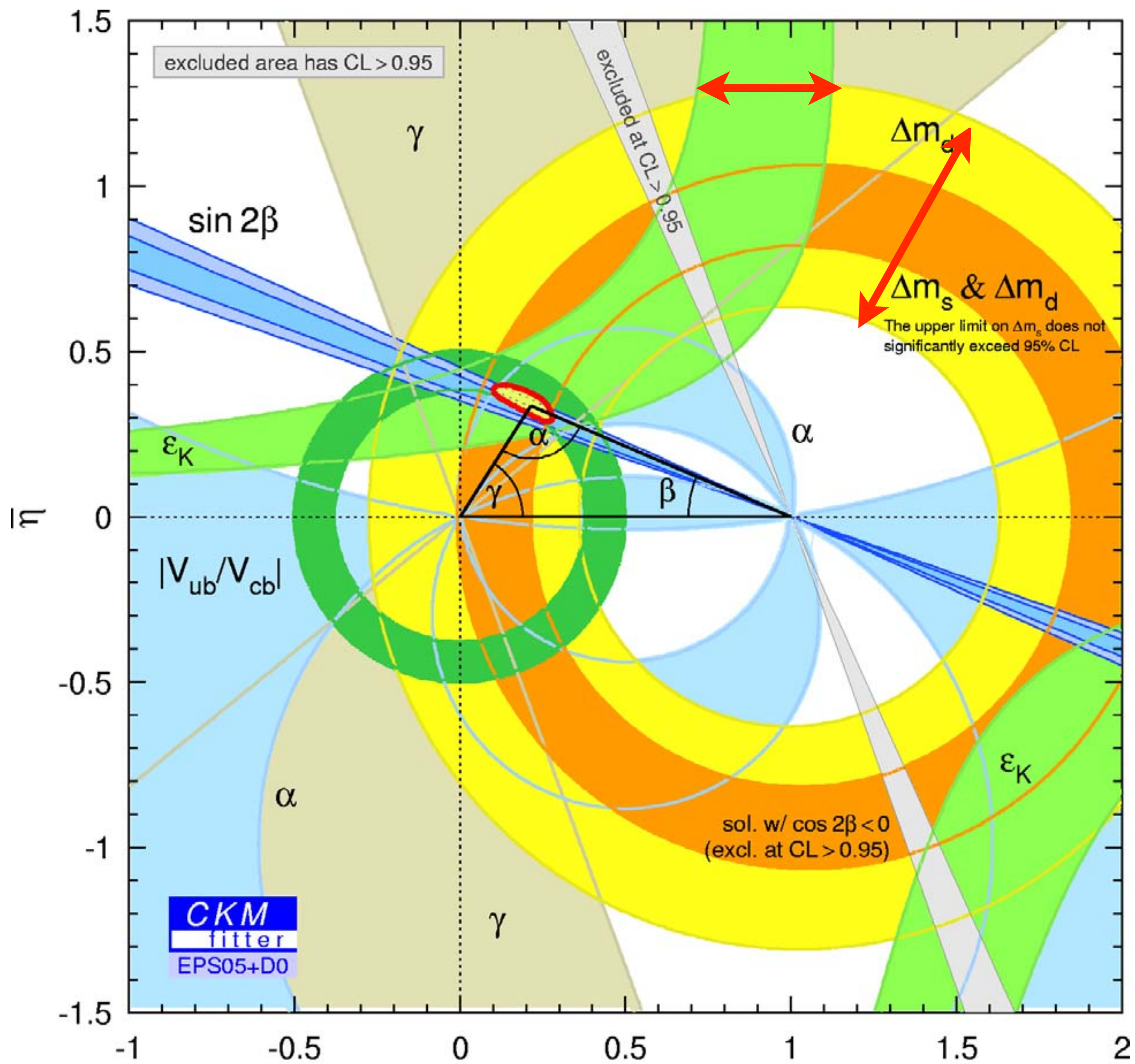
- Staggered/naive
  - Good chiral behavior (can get to light quark masses), but fermion doubling introduces theoretical complications. Cheap.
- Wilson/clover
  - No fermion doubling but horrible chiral behavior.
- Overlap/domain wall
  - Nice chiral behavior at the expense of adding a fifth space-time dimension. Expensive.

The various methods have wildly incommensurate virtues and defects.
Staggered fermion calculations are the cheapest and currently most advanced phenomenologically.

Progress, but also need and opportunity

For some quantities, only lattice calculations can unlock the complete potential of experimental measurements.

$K\overline{K}$ mixing



Bucholz, FPCP 2006

$B\overline{B}$ mixing

$B_s\overline{B}_s$ mixing

Lattice QCD needs to deliver these quantities reliably. Or else.

In this talk...

- Concentrate on lattice CKM physics phenomenology.
  - Unquenched, 2+1 light flavors where possible.
- Concentrate on gold-plated quantities.
  - Other interesting things (order of increasing difficulty)
    - <*B*|*O*|*B*> expectation values for HQET, etc. (Doable now.)
    - $K\pi\pi$. (Doable now, but harder. People are trying.)
    - Broad unstable states. (Being done now, but will be hard to get right.)
    - $B\pi\pi$. (Nobody's trying.)

Thanks, Steve Gottlieb, Richard Hill, Uli Nierste, Masataka Okamoto.
See Okamoto review at Lattice 2005.

Outline

- Introduction
- CKM matrix elements
  - Decay constants
  - $M\overline{M}$ mixing
  - Semileptonic decays
- Outlook

CKM matrix elements

All of the CKM matrix elements except V_{tb} can be determined from one of lattice QCD's golden quantities.

$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \pi \to l\nu & K \to \pi l\nu & B \to \pi l\nu \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ D \to \pi l\nu & D \to K l\nu & B \to D^{(*)} l\nu \\ D \to l\nu & D_s \to l\nu & \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\ \langle B_d | \overline{B}_d \rangle & \langle B_s | \overline{B}_s \rangle & \end{pmatrix}$$

For some, like V_{td} and V_{ts}, lattice calculations are the only road to accurate determinations.

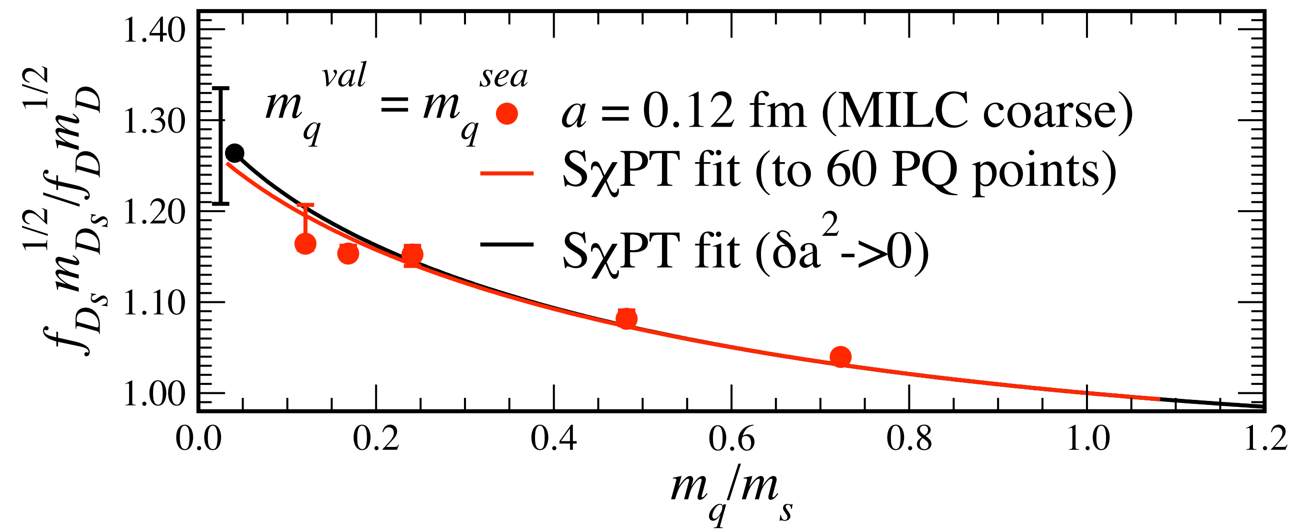
f_D, f_{Ds}

CLEO-c charm physics and the lattice:

- Tests lattice's ablility to accurately calculate amplitudes by producing new measurements of CKM independent quantities that can be checked with the lattice, such as $\frac{\mathcal{B}(D \to l\nu)}{\mathcal{B}(D \to \pi l\nu)}$.

- With good lattice calculations, measures CKM charm matrix elements: V_{cs} and V_{cd}.

f_D, f_{Ds}



$$f_D = 201(03)_{\rm sta}(17)_{\rm sys}\ \mathrm{MeV}$$

$$f_{D_s} = 249(03)_{\rm sta}(16)_{\rm sys}\ \mathrm{MeV}$$

Fermilab/MILC, 05. n_f=2+1 staggered light quarks.
Fermilab heavy quarks.

$$f_D^{n_f=2} = 202(12)_{\rm sta}(^{+20}_{-25})_{\rm sys}$$

$$f_{D_s} = 238(11)_{\rm sta}(^{+07}_{-27})_{\rm sys}\ \mathrm{MeV}$$

CP-PACS, 05. n_f=2 clover light quark.
“RHQ” heavy quarks.

Compare with CLEO-c

CLEO error dominated by statistics, will be reduced with full data set.

Assumes canonical V_{cd}.

f_{D+} = (223±17±3) MeV

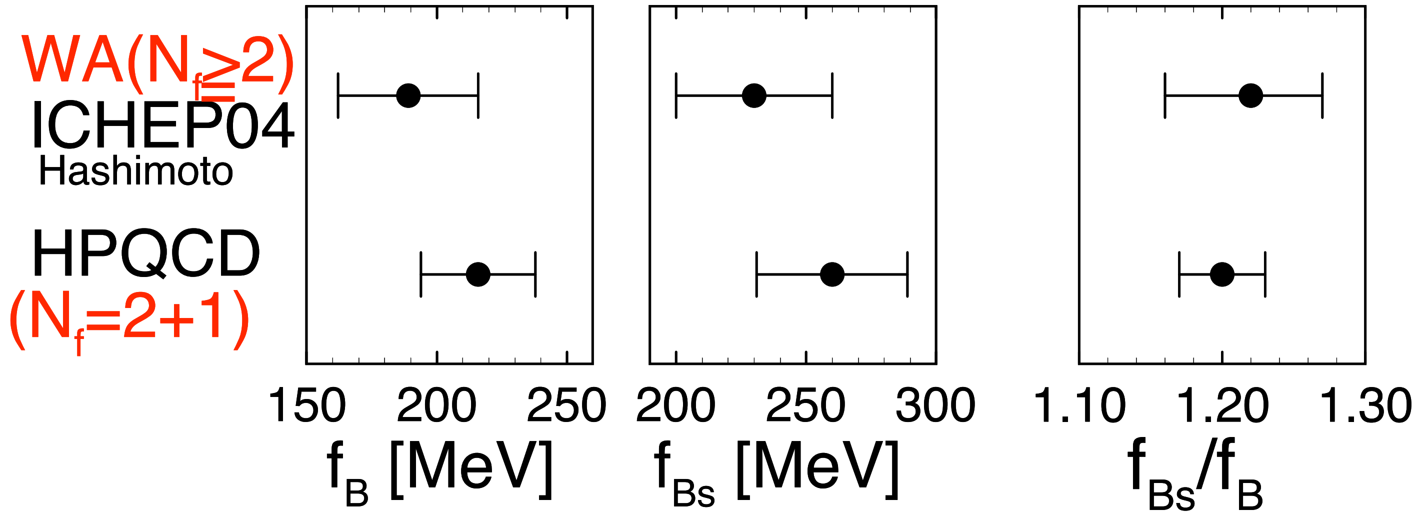
f_{D+} = (201±3±17) MeV

LQCD (PRL 95 251801, ’05)

Lattice error dominated by discretization error (done on a single lattice spacing).
Will be reduced by in progress calculations on multiple lattice spacings.

CLEO-c. R. Poling, FPCP 2006.

f_B, f_{Bs}



Okamoto, Lattice 2005

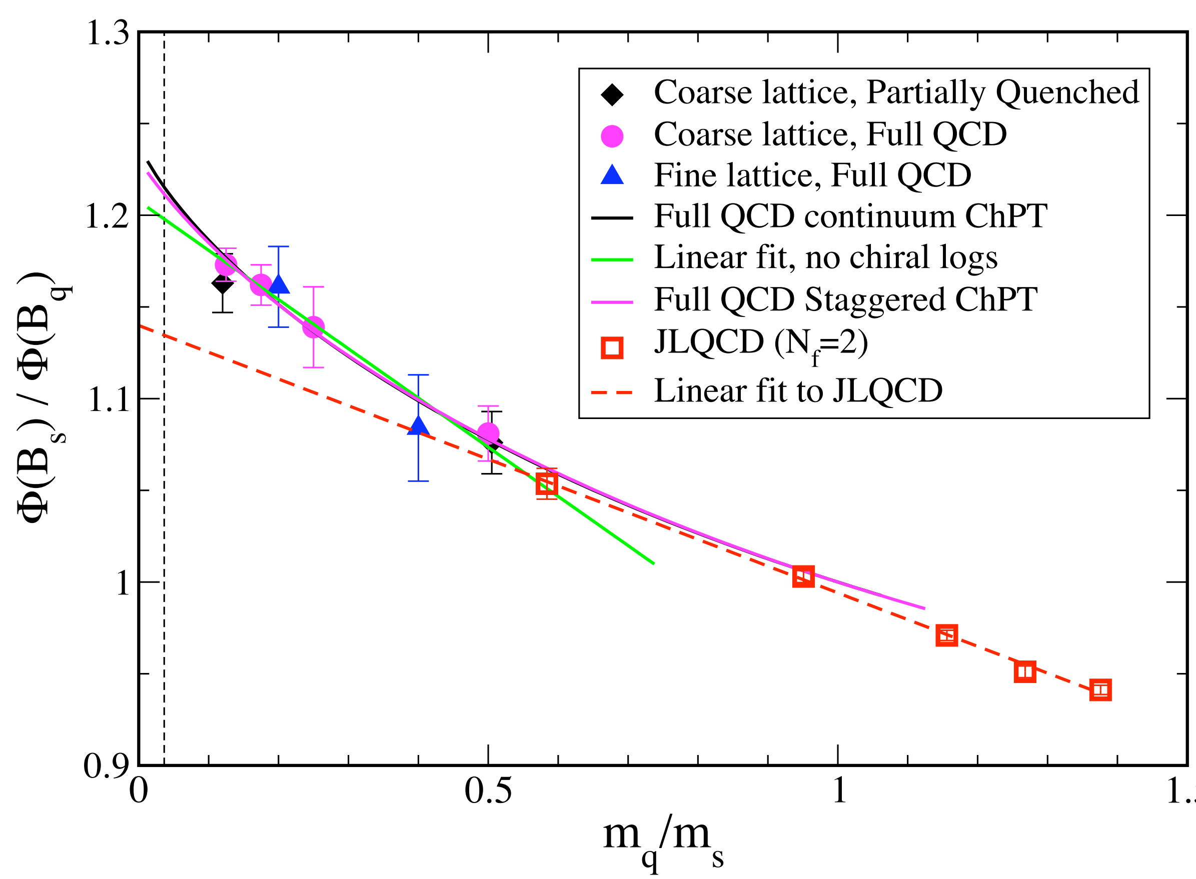
HPQCD 05. n_f=2+1 staggered light quarks, NRQCD heavy quarks.

Dominant uncertainty in f_B:
$O(\alpha^2)$ perturbation theory.

Dominant uncertainty in f_{Bs}/f_B:
Statistics and chiral extrapolation.

$$f_{B_s}/f_B = 1.20(3)_{\mathrm{sta}+\chi\mathrm{fit}}(1)_{\mathrm{others}}$$

PT error cancel $\Longrightarrow$ total 3%



f_B, f_{Bs}

Using $|V_{ub}| = (4.38 \pm 0.33) \times 10^{-3}$ from HFAG

Compare with new Belle result for f_B:

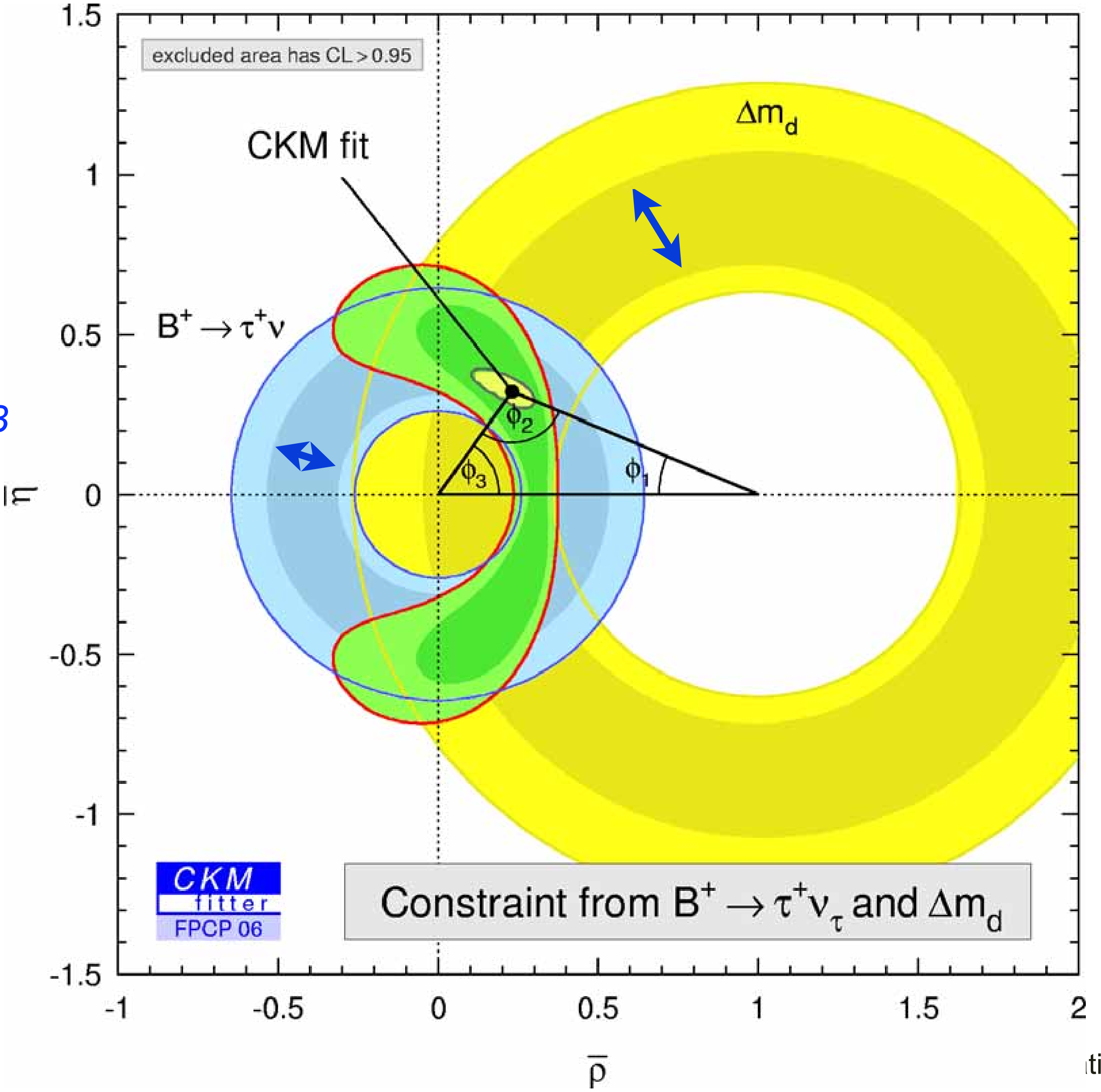
$$f_B = 0.176^{+0.028}_{-0.023}(\mathrm{stat})^{+0.020}_{-0.018}(\mathrm{syst})\ \mathrm{GeV}$$

$f_B = 0.216 \pm 0.022$ GeV (HPQCD)

Phys. Rev. Lett. 95, 212001 (2005)

CKM constraint is fit using $B \to \tau\nu / \Delta M_d$.
(f_B drops out.)

Much tighter constraints can be obtained by incorporating lattice f_B and B_B (<15%).



Ikado, FPCP 2006

f_K, f_π

$$f_\pi = 128.1 \pm 0.5 \pm 2.8 \text{ MeV}\,, \qquad f_K = 153.5 \pm 0.5 \pm 2.9 \text{ MeV}$$

MILC 05. n_f=2+1 staggered.

Light quark masses essential.

$$f_K/f_\pi = 1.198(3)(^{+16}_{-\ 5})$$

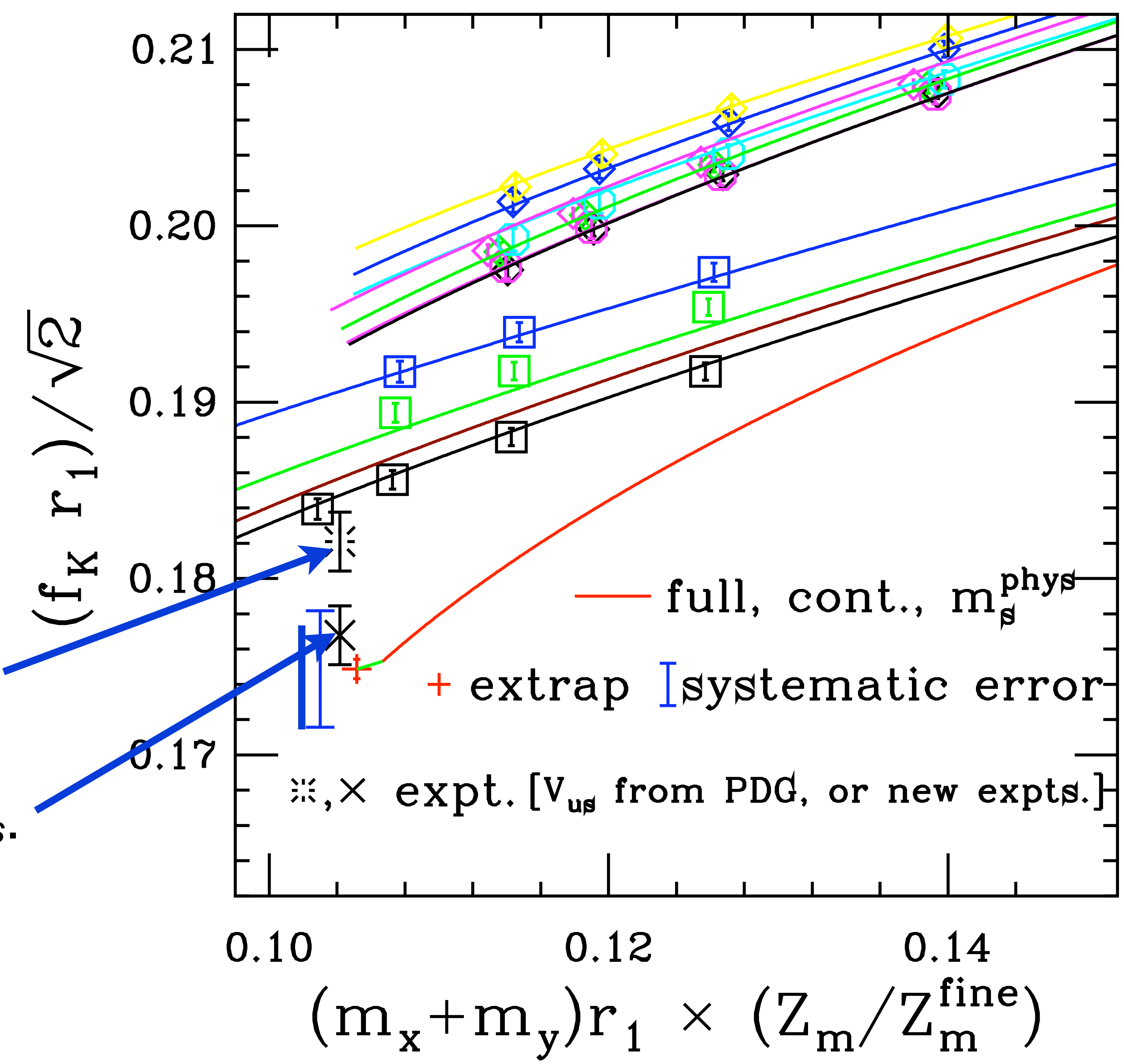
⇨ $|V_{us}| = 0.2242(^{+11}_{-31})$

(cf. 0.2200(26) (old);
0.2262(23) (new).)

Chiral extrapolation of f_K.

Leptonic decay experiment + “old” V_{us}.

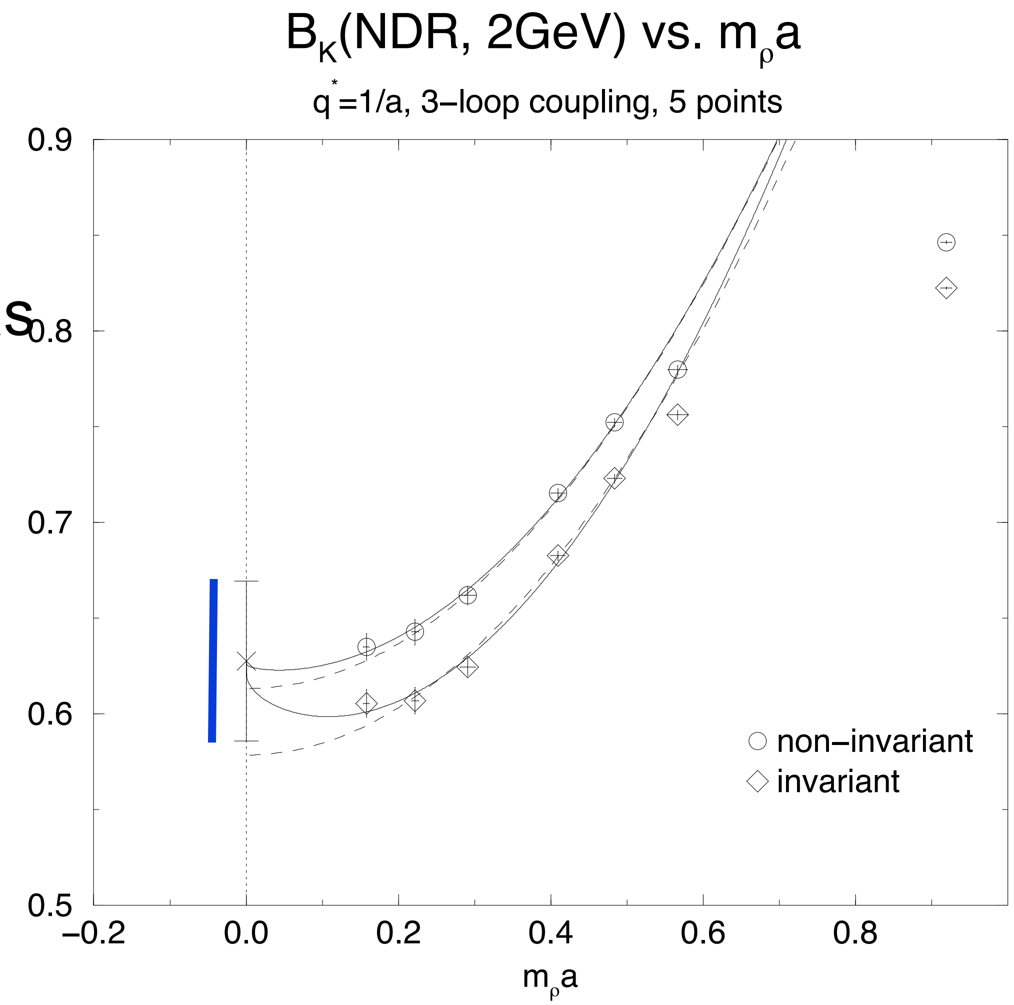
Leptonic decay experiment + “new” V_{us}.

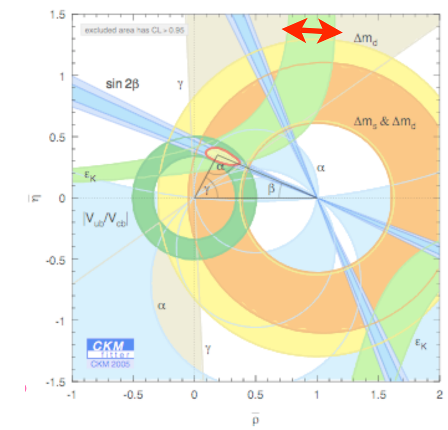


$K\bar{K}$ Mixing

Benchmark calculation for years was JLQCD 97, staggered fermions. Quenched!

$$B_K(NDR, 2\text{GeV}) = 0.628(42)$$





Tons of CPU power to get to light quark masses.

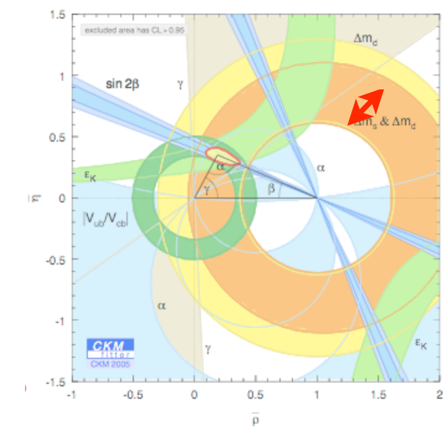
Now a field of hot activity

Lots of investigations of new methods.

At least two 2+1 programs started

See Dawson, Lattice 2005.

$B\bar{B}$ Mixing



$$Q = \bar{q}_L \gamma_\nu b_L \, \bar{q}_L \gamma^\nu b_L$$

$$\langle \bar{B}^0 | (\bar{b}q)_{V-A} (\bar{b}q)_{V-A} | B^0 \rangle \propto B_{B_q} f_{B_q}^2$$

$$\Delta M_{B_{d(s)}} \propto B_{B_{d(s)}} f_{B_{d(s)}}^2 \; |V_{tb}^* V_{td(s)}|^2$$

$$B(m_b) = 0.836(27)\left({}^{+56}_{-62}\right) \quad , \quad \hat{B}_s / \hat{B} = 1.017(16)\left({}^{+56}_{-17}\right)$$

JLQCD, 03
nf=2 clover light,
NRQCD heavy quarks.

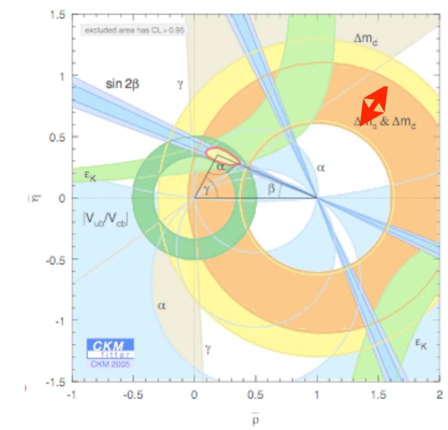
Combine with HPQCD f_B to obtain:

$$f_B \sqrt{\hat{B}_B} \;=\; 244(26)\text{MeV}$$

$$|V_{td}|_{\text{Lat05}} = 7.4(0.8)\times 10^{-3})$$

$$(|V_{td}|_{\text{PDG04}} = 8.3(1.6)\times 10^{-3})$$

$B_s\bar{B}_s$ Mixing



D. Bucholz, FPCP06

D0: $17<\Delta m_s<21$ ps-1 @90% CL; 2.3σ

CDF: Talk by Guillelmo Gomez-Ceballos, 3PM, today.

Combining

$$f_{B_s}/f_B = 1.20(3)_{\mathrm{sta}+\chi\mathrm{fit}}(1)_{\mathrm{others}}$$

PT error cancel $\Longrightarrow$ total 3%

HPQCD

and

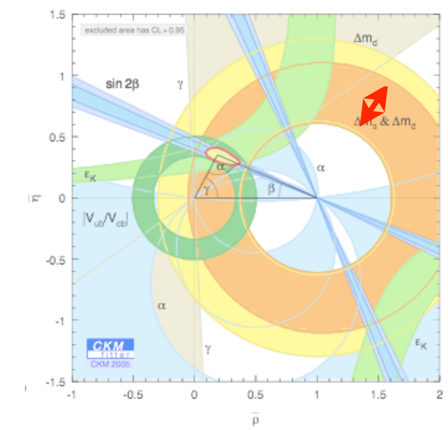
$$\hat{B}_s/\hat{B} = 1.017(16)(^{+56}_{-17})$$

JLQCD

Okamoto obtains

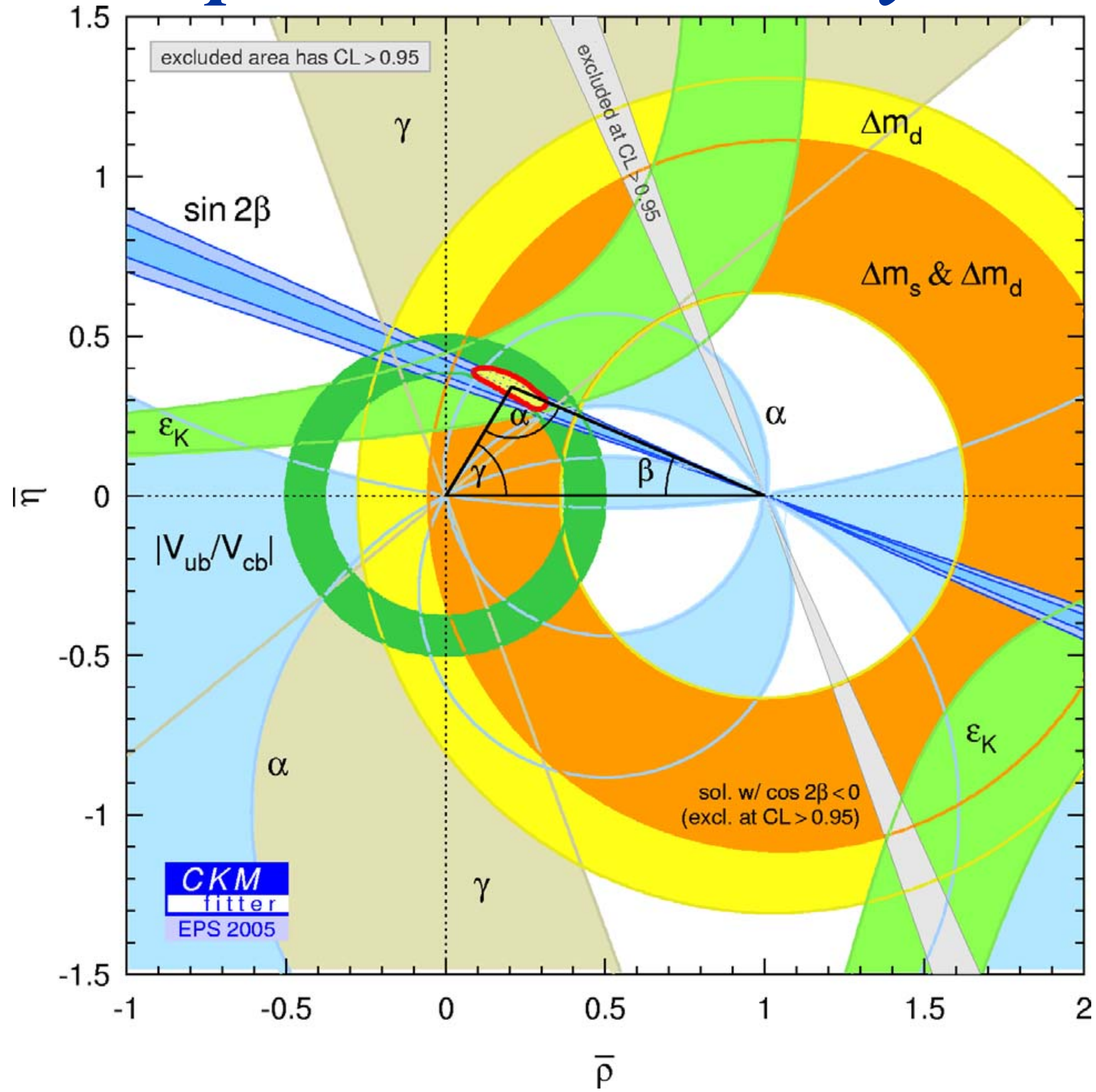
$$f_{B_s}/f_B\sqrt{\hat{B}_{B_s}/\hat{B}_B} \quad = \quad 1.210(^{+47}_{-35})$$

$B_s\bar{B}_s$ Mixing



Effect of D0 result on CKM fits:

Impact on the Unitarity Triangle



D. Buchholz

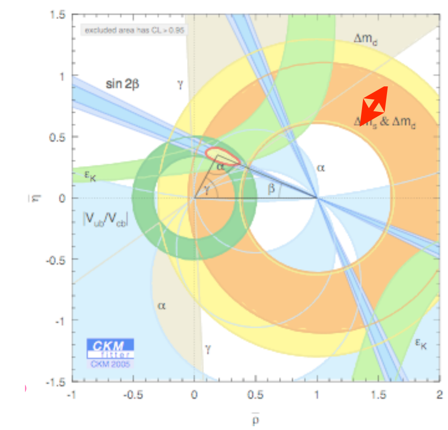
Cut and pasted from Okamoto's Lattice 2005 review transparencies:

$$f_{B_s}/f_B\sqrt{\hat{B}_{B_s}/\hat{B}_B} \;=\; 1.210(^{+47}_{-35})$$

$\delta(|V_{td}|/|V_{ts}|) = 3 - 4\%$

with forthcoming ΔM_{B_s}

$B_s\bar{B}_s$ Mixing



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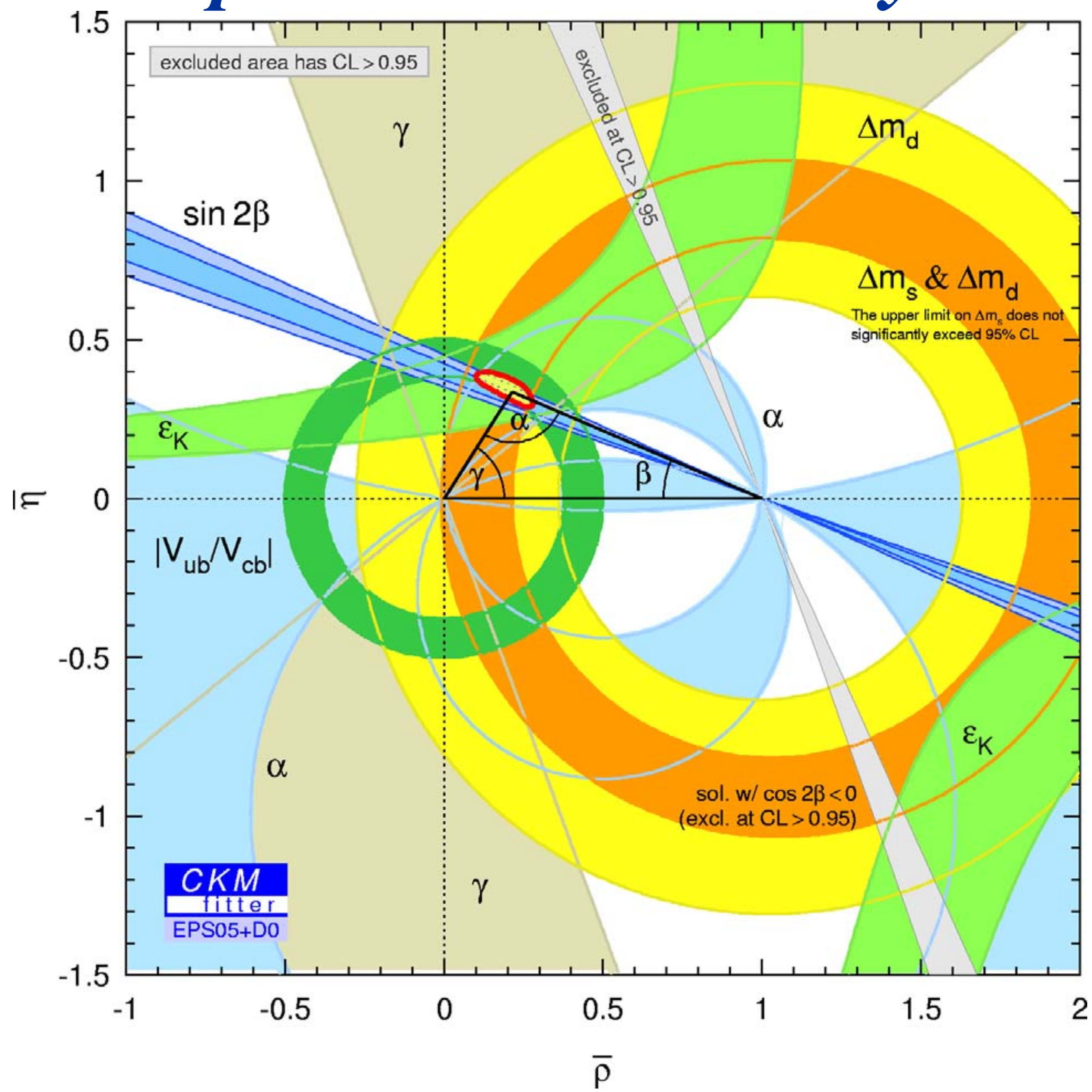

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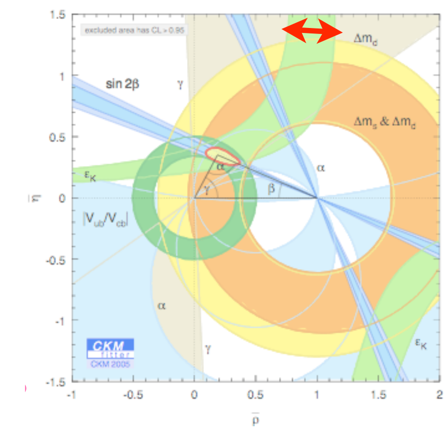
$B_s \bar{B}_s$ Mixing



$$\Delta\Gamma_s = 0.097 \begin{array}{l} +0.041 \\ -0.042 \end{array} \text{ ps}^{-1}$$

Van Kooten
FPCP 2006

$$\bar{\tau} = \frac{1}{\Gamma_s} = 1.461 \pm 0.030 \text{ ps}$$

New operator needed: $Q_S = \bar{b}_R s_L \, \bar{b}_R s_L$

Not done unquenched, but Becirevic et al., 01, have calculated the complete set of four-quark operators quenched:

$$\begin{aligned}
O_1 &= \bar{b}^i \gamma_\mu (1-\gamma_5) q^i \, \bar{b}^j \gamma_\mu (1-\gamma_5) q^j \,, \\
O_2 &= \bar{b}^i (1-\gamma_5) q^i \, \bar{b}^j (1-\gamma_5) q^j \,, \\
O_3 &= \bar{b}^i (1-\gamma_5) q^j \, \bar{b}^j (1-\gamma_5) q^i \,, \\
O_4 &= \bar{b}^i (1-\gamma_5) q^i \, \bar{b}^j (1+\gamma_5) q^j \,, \\
O_5 &= \bar{b}^i (1-\gamma_5) q^j \, \bar{b}^j (1+\gamma_5) q^i \,,
\end{aligned}$$

$$\begin{aligned}
B_1^{(d)\overline{\rm MS}}(m_b) &= 0.87(4)(3)(0)\left(^{+4}_{-2}\right), \, B_1^{(s)\overline{\rm MS}}(m_b) = 0.87(2)(3)(0)\left(^{+4}_{-2}\right), \\
B_2^{(d)\overline{\rm MS}}(m_b) &= 0.83(3)(3)(1)(2) \,, \, B_2^{(s)\overline{\rm MS}}(m_b) = 0.84(2)(3)(1)(2) \,, \\
B_3^{(d)\overline{\rm MS}}(m_b) &= 0.90(6)(3)(7)(2) \,, \, B_3^{(s)\overline{\rm MS}}(m_b) = 0.91(3)(3)(7)(2) \,, \\
B_4^{(d)\overline{\rm MS}}(m_b) &= 1.15(3)(4)\left(^{+0}_{-4}\right)(3) \,, \, B_4^{(s)\overline{\rm MS}}(m_b) = 1.16(2)(4)\left(^{+0}_{-4}\right)(3) \,, \\
B_5^{(d)\overline{\rm MS}}(m_b) &= 1.72(4)(5)\left(^{+19}_{-00}\right)(3) \,, \, B_5^{(s)\overline{\rm MS}}(m_b) = 1.75(3)(5)\left(^{+20}_{-00}\right)(3) \,,
\end{aligned}$$

Now must be repeated, unquenched.

$B \to D l \nu$

Form factor shape is well-measure in experiment.
Theory must supply the normalization.

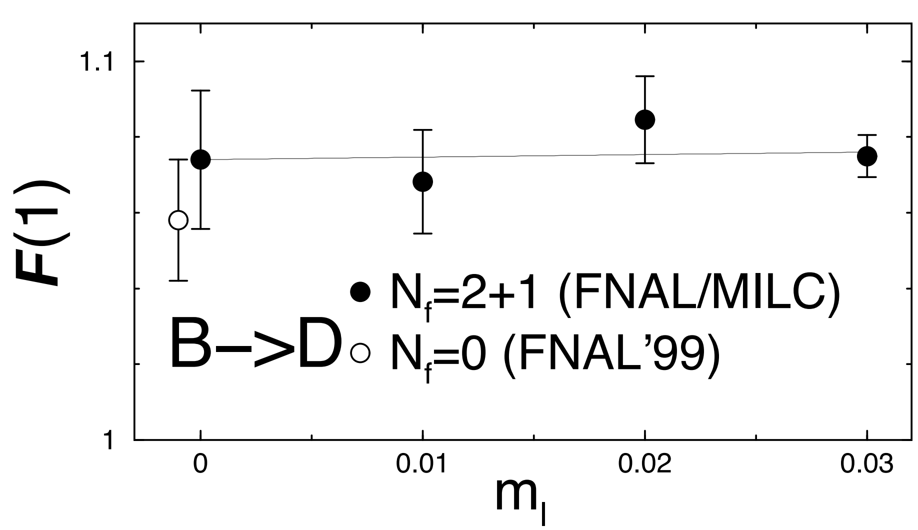
Heavy quark theory: normalization $\to$1 in the HQ symmetry limit.

But, high precision is required.

Ratio method: determine the form factor from a quantity which goes to 1 with vanishing errors in the symmetry limit.

$$\frac{\langle D|V_0|B\rangle\langle B|V_0|D\rangle}{\langle D|V_0|D\rangle\langle B|V_0|B\rangle}$$

Fermilab 99.



$$\mathcal{F}_{B\to D}(1) = 1.074\ (18)_{\rm sta}(15)_{\rm sys}$$

Using HFAG'04 avg for $|V_{cb}|\mathcal{F}(1)$,

$$|V_{cb}|_{\rm Lat05} = 3.91(09)_{\rm lat}(34)_{\rm exp}\times 10^{-2}$$

Fermilab/MILC 05.

$K \to \pi l \nu$

Similar situation. Amplitude is normalized to1 in the (chiral) symmetry limit.

Rome (Becirevic et al.) 04: try the same approach, the ratio method.

$f_+(0)$:

Leutwyler-Roos	quark model	0.961(8)
Becirevic et al.	n_f=0	0.960(5)(6)
JLQCD	n_f=2	0.952(6)
Fermilab/MILC	n_f=2+1	0.962(6)(9)
RBC	n_f=2	0.964(9)(5)

No surprises from theory.
Kl3 experiment explains the first row unitarity puzzle.

$D \to \{K, \pi\} l\nu$

CLEO-c/lattice charm physics goals:
- Test lattice amplitude calculations on CKM independent combinations of amplitudes.
- Use tested lattice calculations to obtain new CKM determinations.

Test lattice:

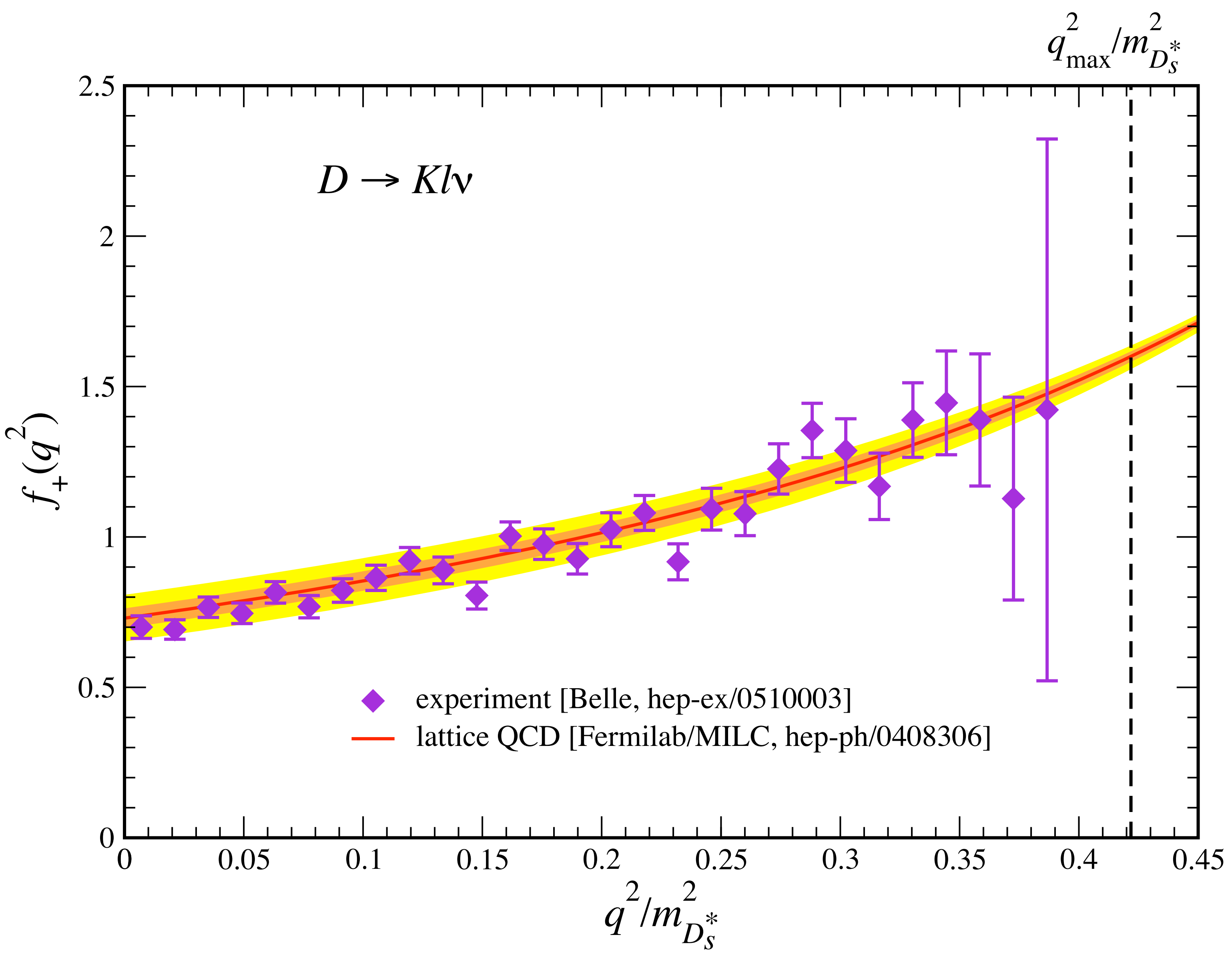
$$R_{cd} \equiv \sqrt{\frac{\mathcal{B}(D \to l\nu)}{\mathcal{B}(D \to \pi l\nu)}} \propto \frac{f_D}{f_+^{D\to\pi}(0)} \cdot \frac{|\cancel{V}_{\mathbf{cd}}|}{|\cancel{V}_{\mathbf{cd}}|}$$

R_{cd}=0.22(2) Fermilab/MILC n_f=2+1

R_{cd}=0.25(2) CLEO-c

$D \to \{K, \pi\} l\nu$

A prediction: shape of the $D \to K\, l\nu$ form factor.



CLEO-c is threatening to drastically improve. → More stringent tests.

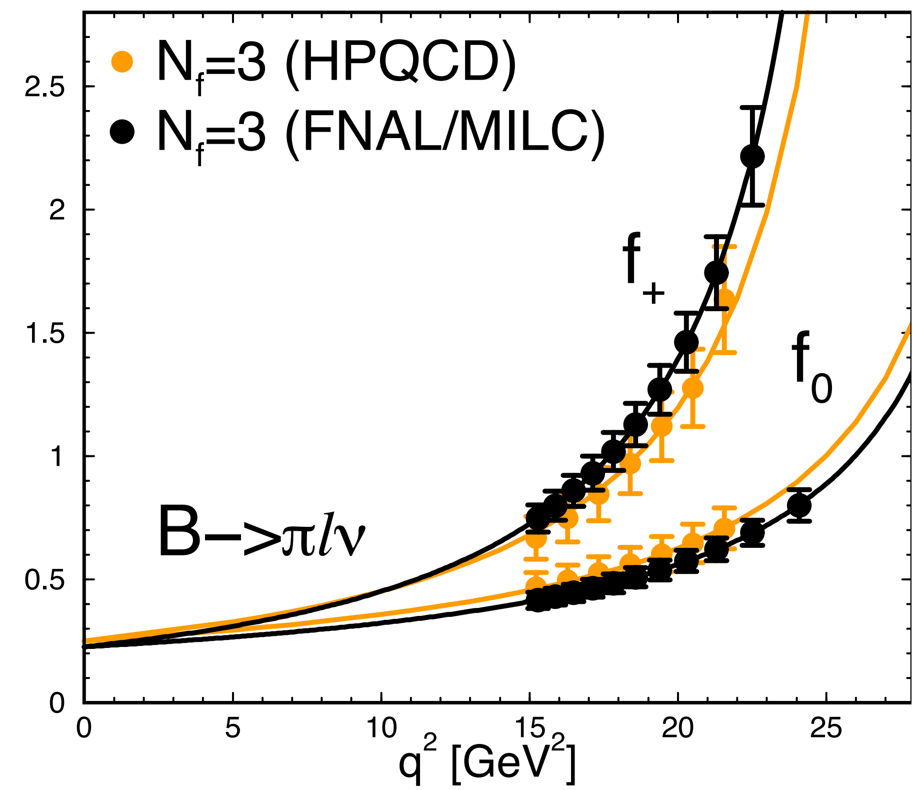
$D \to \{K, \pi\} l\nu$

Apply: determine CKM elements.

Decay Mode	$\lvert V_{cx} \rvert \pm (stat) \pm (syst) \pm (theory)$	PDG (HF) Value
$D^0 \to \pi^{\pm} e \nu$	$0.221 \pm 0.013 \pm 0.004 \pm 0.028$	0.224 ± 0.012
$D^0 \to K^{\pm} e \nu$	$1.006 \pm 0.042 \pm 0.013 \pm 0.103$	0.996 ± 0.013 (0.976 ± 0.014)
$D^{\pm} \to \pi^0 e \nu$	$0.235 \pm 0.016 \pm 0.006 \pm 0.029$	0.224 ± 0.012
$D^{\pm} \to K^0 e \nu$	$0.984 \pm 0.042 \pm 0.017 \pm 0.101$	0.996 ± 0.013 (0.976 ± 0.014)

CLEO-c. R. Poling, FPCP 2006.

$B \to \pi l \nu$



Lattice data cover on 1/3 of physical q^2 range. More challenging to compare with experiment than anything else covered in this talk.

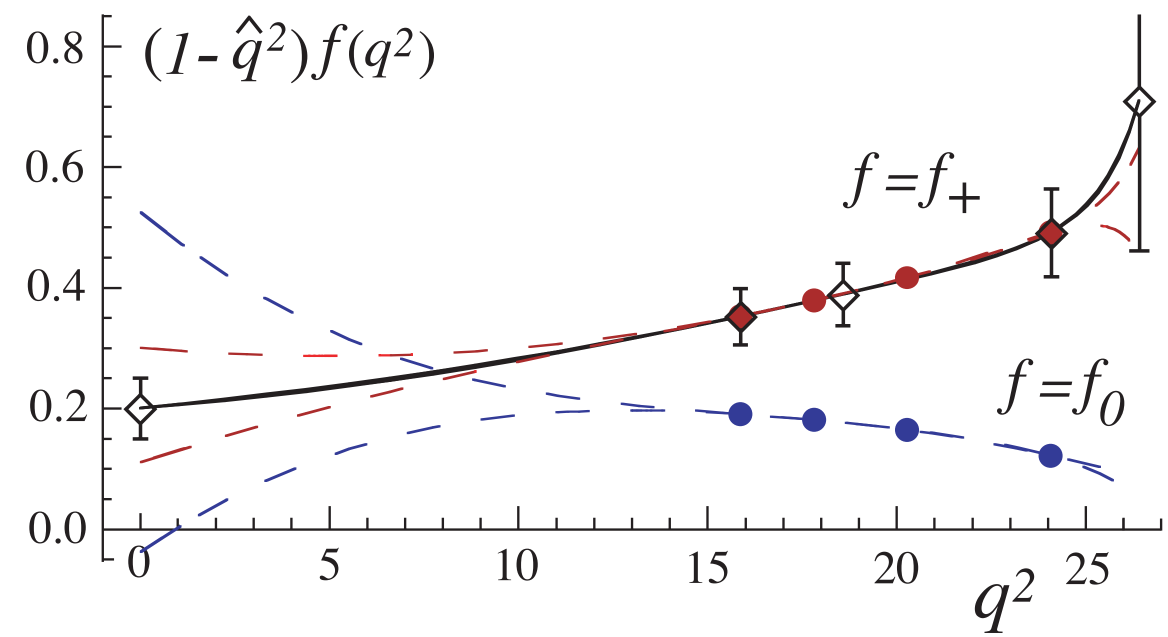
I'll discuss here how to go beyond current methods, rather than current results.

Approaches:

- Moving NRQCD (HPQCD)
- Add SCET point at q^2 =0 (Arnesen et al.)

Arnesen, Rothstien, Grinstien, and Stewart add SCET point at q^2=0 to lattice data, use unitarity and analyticity to bound form factor.

What do unitarity and anlyticity alone say?



$B \to \pi l \nu$

The function $$z(t,t_0) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$ ($t = q^2 = (p_H - p_L)^2$, $t_+ = (m_H + m_L)^2$, $t_- = (m_H - m_L)^2$).

maps the physical q^2 region into

$B \to \pi l \nu$: $-0.34<z<0.22$,
$D \to \pi l \nu$: $-0.17<z<0.16$,
$D \to K l \nu$: $-0.04<z<0.06$,
$B \to D l \nu$: $-0.02<z<0.04$.

The form factors can be written

$$f(t) = \frac{1}{P(t)\phi(t,t_0)} \sum_{k=0}^{\infty} a_k(t_0)\, z(t,t_0)^k$$

Accounts for B* pole.

Calculable function to make a_ks look simple.

Unitarity requires just that $$\sum_{k=0}^{n_A} a_k^2 \le 1$$

According to the unitarity bound, even for $B \to \pi l \nu$, 5 or 6 terms in series suffice for 1% accuracy.

$B \to \pi l \nu$

Becher and Hill (Richard Hill talk, later this morning): In

$$f(t) = \frac{1}{P(t)\phi(t,t_0)} \sum_{k=0}^{\infty} a_k(t_0)\, z(t,t_0)^k$$

$$\sum_{k=0}^{\infty} a_k^2 \quad \text{of order } (\Lambda/m_b)^3$$

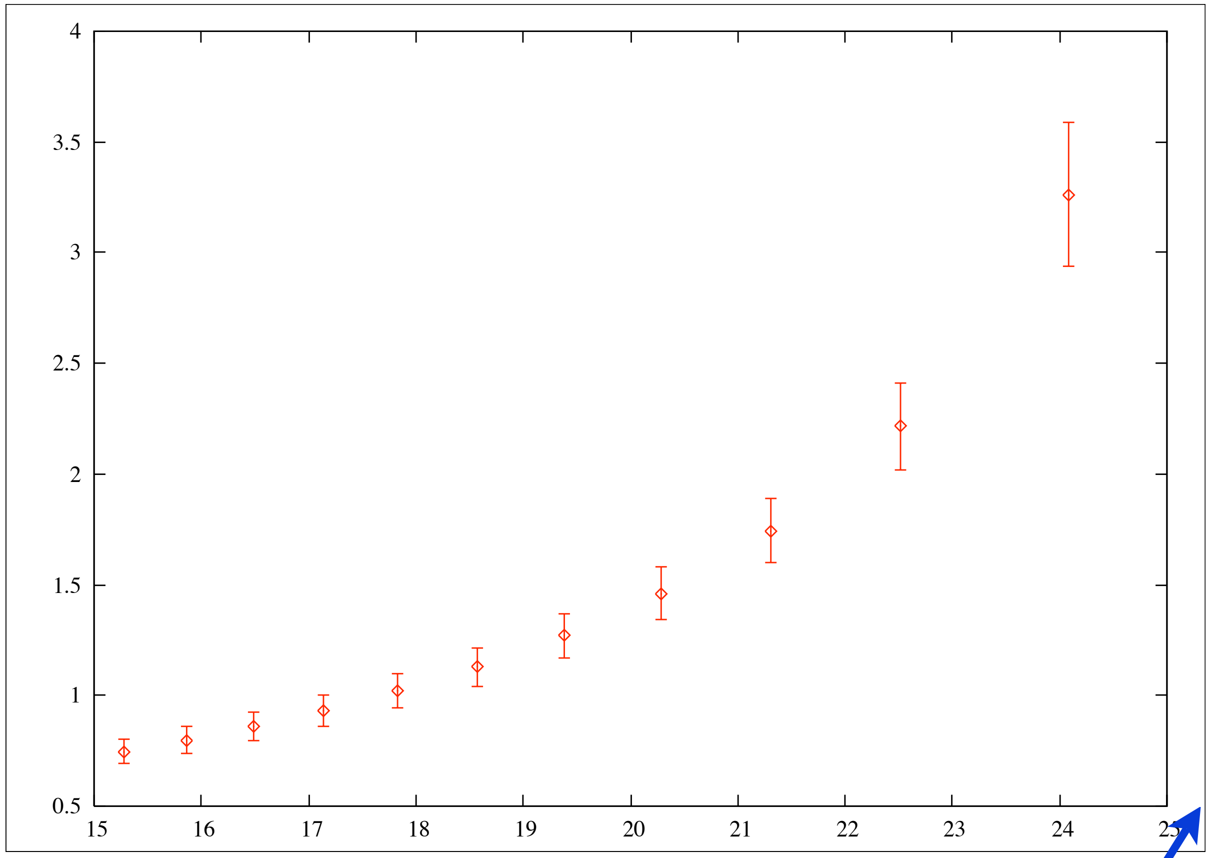
Two (maybe three) terms should suffice in power series for 1% accuracy in form factors.

Test on lattice results:

Fit Fermilab/MILC lattice data to Z expansion:

Our form factor data for $B \to \pi l \nu$, chirally extrapolated.

$f_+(q^2)$

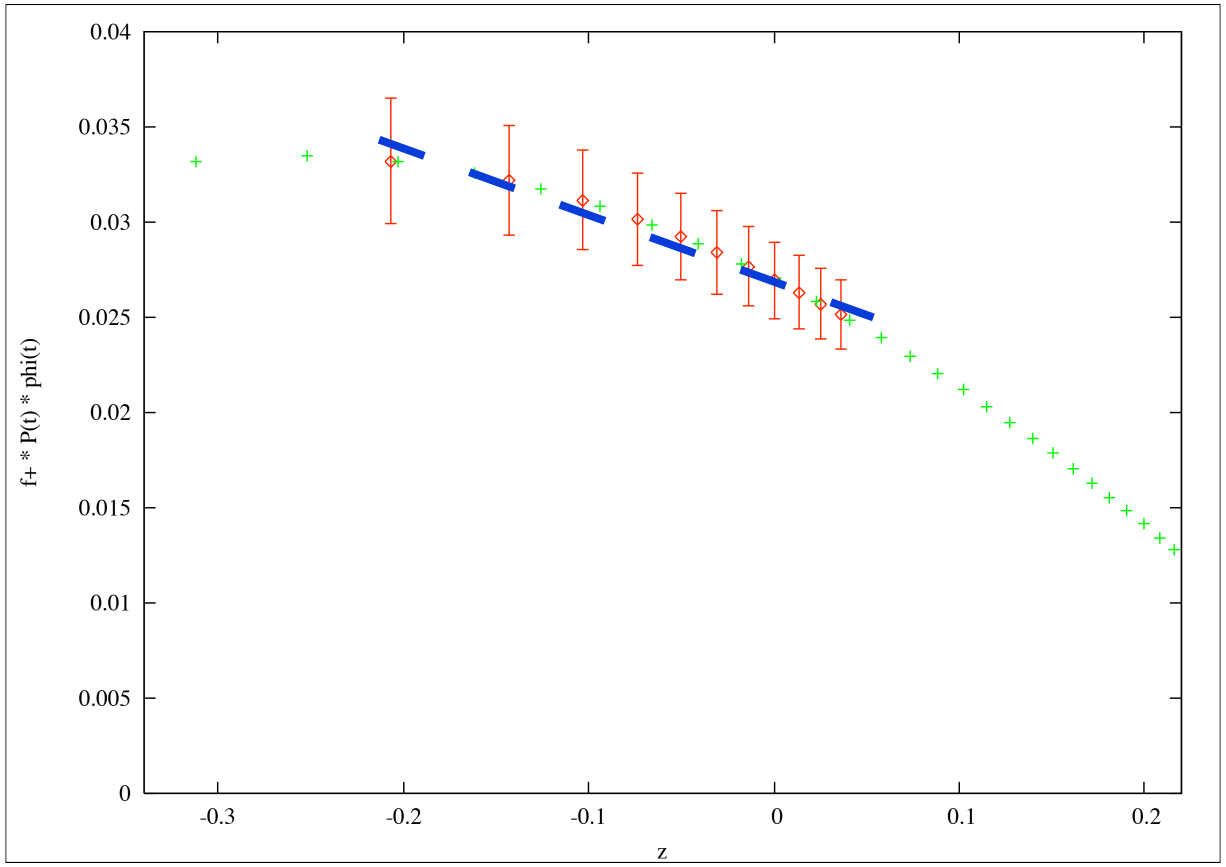


q^2 q^2_{max}

Replot of our data vs. z and fit with P and phi removed.

$$\sum_{k=0}^{\infty} a_k(t_0)\, z(t,t_0)^k$$

Red, our lattice data; green, fit.



Normalization plus slope fits fine!

$B \to \pi l \nu$

Upshot:

If Becher and Hill are right, comparing shapes between theory and experimental form factors could be almost as simple for $B \to \pi l \nu$ as for $B \to D l \nu$ and $K \to \pi l \nu$:

1) Measure normalization and slope,
2) Search for evidence of curvature.

Crucial to use the right variables, though.

Outlook: rich ferment in simulation methods

- Many new ideas in the last few years.
  - Domain-decomposition (Lüscher; ...).
  - Rational hybrid Monte Carlo (Clark and Kennedy; ...).
  - Short-, long-scale separation (Peardon and Sexton; ...).
  - ...
- This makes possible
  - Big overall speedups.
  - Closer approach to the chiral limit for Wilson fermions.

Outlook: computing

The US currently has about 10 Teraflops of delivered CPU power devoted to lattice QCD (QCDOCs at BNL and clusters at Fermilab and JLab); adding on at a rate of $2M/year.

Large new installations of lattice computing are planned throughout the world.

Location	type	size	peak	est. perf.	total
Paris-Sud	apeNEXT	1 racks	0.8 TF	0.4 TF	0.4
Bielefeld	apeNEXT	6 (3) racks	4.9 TF	2.5 TF	
DESY (Zeuthen)	apeNEXT	3 racks	2.5 TF	1.2 TF	
Julich	BlueGene/L	8 racks	45.8 TF	11.5 TF $\times 1/2$?	10–15
Munich	SGI Tollhouse	3328 nodes	70 TF	14 TF?? $\times$?	
Rome	apeNEXT	12 (8) racks	9.8 TF	4.9 TF	5
KEK	BlueGene/L	10 racks	57.3 TF	14.3 TF	
Tsukuba	PACS-CS	2560 nodes	14.3 TF	3.3 TF	14–18
KEK	Hitachi		2.1 TF	1 TF ?	
Edinburgh	QCDOC	12 racks	9.8 TF	4.2 TF	4–5
Edinburgh	BlueGene/L	1 racks	5.7 TF	1.4 TF $\times$?	

~50 TF planned.

Steve Gottlieb

Outlook: simulation projects

All of the major methods for lattice fermions will be under serious investigation somewhere in the world.

- KEK Blue Gene: overlap.
- Tsukuba PACS-CS: Wilson-clover.
- Julich Blue Gene: overlap, twisted mass.
- DESY, Paris, Rome apeNEXT: twisted mass.
- BNL/Edinburgh QCDOC: domain wall.
- US QCDOC/clusters: improved staggered.

Summary

- There is currently more activity and progress in methods and algorithms than there has been since 20 years ago.
- 10s of teraflops in CPU power devoted to lattice QCD are now coming on line.
- Many of the most important results for phenomenology are among the cleanest lattice calculations (such as pseudoscalar meson decay constants and mixings).

We're in a period of rapid development for lattice QCD that shows no signs of slowing down.