Molecular Quarkonium

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- Introduction.

General remarks on molecular quarkonium.

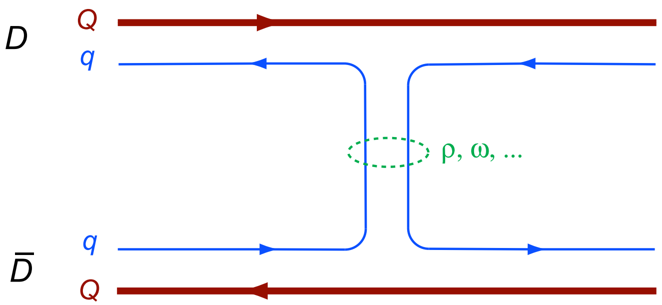
- X(3872)
  - Essential known properties
  - Overview of the models
  - ‘Peripheral’ and ‘core’ components
  - Spin selection rule and production/decay
  - ‘Peripheral’ decays to $D^0\bar{D}^0\pi^0$ and $D\bar{D}\gamma$
- “$\psi(4040)$”

Channel coupling and angular correlations.

- Other molecules (?)
- Summary

Introduction

- A simple idea:



A normal "nuclear" force. The strength and the radius r_0 do not depend on m_Q.

$$E = \frac{p_D^2}{m_D} + U(r)$$

At sufficiently large m_Q $(m_D = m_Q + \bar{\Lambda} + \ldots)$ the $U(r)$ wins.

Heavy-meson bound states, "molecules" should exist. L.B. Okun & MV '76

However putative "molecules" can be quite generic.
Simple model: ρ and ω exchange.

$$\rho = (\bar{u}u - \bar{d}d)/\sqrt{2}\,. \quad \text{Coupling :} \;\; \sqrt{2}\, g_\rho \, \rho^a T^a \,.$$

$$\omega = (\bar{u}u + \bar{d}d)/\sqrt{2}\,. \quad \text{Coupling :} \;\; \frac{g_\omega}{\sqrt{2}}\, \omega \,.$$

Net interaction potential in a state with isospin I: $U \propto g_\rho^2 \left(I(I+1) - \frac{3}{2} \right) - g_\omega^2$
$\Rightarrow U_{I=0} \propto -3g_\rho^2 - g_\omega^2 \,, \qquad U_{I=1} \propto g_\rho^2 - g_\omega^2 \,.$

In VDM $g_\rho^2 = g_\omega^2 \Rightarrow$ attraction only in the $I = 0$ channel (and possibly sufficient for binding $D^{(*)}\bar{D}^{(*)}$ in the S wave).

However hard to distinguish $I = 0$ molecule $(Q\bar{q})(\bar{Q}q)$ from quarkonium $\bar{Q}Q$, especially in the S wave:

$D\bar{D}$:	0^{++}	$({}^3P_0\ \bar{Q}Q)$
$D\bar{D}^* + \bar{D}D^*$:	1^{++}	$({}^3P_1\ \bar{Q}Q)$
$D\bar{D}^* - \bar{D}D^*$:	1^{+-}	$({}^1P_1\ \bar{Q}Q)$
$D^*\bar{D}^*$:	0^{++}	$({}^3P_0\ \bar{Q}Q)$
	1^{++}	$({}^3P_1\ \bar{Q}Q)$
	2^{++}	$({}^3P_2\ \bar{Q}Q)$

A P wave "crypto-molecule" charmonium was discussed in relation to $\psi(4040)$.

(Then called $\psi(4028)$.)

A. De Rujula, H. Georgi, S. Glashow '77

At $\psi(4040)$:

$$\sigma(D^*\bar{D}^*) : \sigma(D^*\bar{D} + D\bar{D}^*) : \sigma(D\bar{D}) \approx 11 : 8 : 1$$

inspite of the ratio of p^3:

$$p^3(D^*\bar{D}^*) : p^3(D^*\bar{D} + D\bar{D}^*) : p^3(D\bar{D}) \approx 1 : 15 : 38$$

$\psi(4040) = D^*\bar{D}^*$ molecule? (Moot because of $J^{PC} = 1^{--}$)

(Will return to discussion of $\psi(4040)$.)

In $D\bar{D}^* \pm \bar{D}D^*$ and $D^*\bar{D}^*$ systems a π exchange (longer range) is possible. $\Rightarrow$ If the π exchange is crucial for binding, no $D\bar{D}$ molecules , only $D\bar{D}^* \pm \bar{D}D^*$ and/or $D^*\bar{D}^*$.

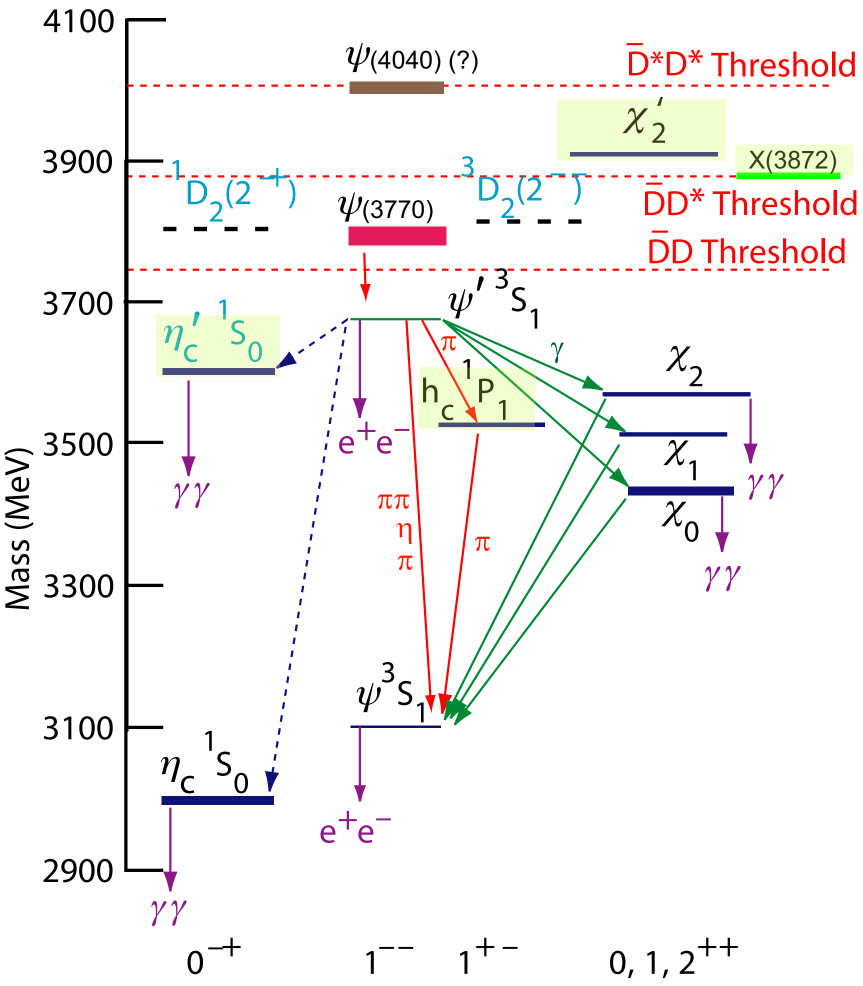
"deusons" N.A. Törnqvist '91.

X(3872)

'03-'04. Belle: $B \to K\, J/\psi\, \pi^+\pi^-$, CDF, $D\emptyset$: $p\bar{p} \to J/\psi\, \pi^+\pi^- + X$. Resonance $X(3872)$ in the $J/\psi\, \pi^+\pi^-$ channel.

Belle, CDF: 10 - 11 σ. Also observed by BaBar.

Mass $3871.7 \pm 0.6\, MeV$ is within less than 1 MeV from the $D^0 \bar{D}^{0*}$ threshold ($3871.2 \pm 0.5\, MeV$). Width unresolved (less than $\sim$ 2 MeV).



- Essential properties of the X(3872) (so far)
  - Small width. $\Gamma <\sim 2MeV$ (exp. resolution). (Compare with $\Gamma(\psi(3770)) \to D\bar{D} \approx 26\,MeV$.) $\Rightarrow$ Unnatural J^P.
  - $X(3872) \to \pi^+\pi^-\pi^0\,J/\psi$ (actually $X(3872) \to \omega\,J/\psi$); Belle hep-ex/0505037

$$\frac{\Gamma(X(3872) \to \pi^+\pi^-\pi^0\,J/\psi)}{\Gamma(X(3872) \to \pi^+\pi^-\,J/\psi)} = 1.0 \pm 0.4 \pm 0.3$$

  - $X(3872) \to \gamma\,J/\psi \;\Rightarrow\; C = +1$; Belle hep-ex/0505037

$$\frac{\Gamma(X(3872) \to \gamma\,J/\psi)}{\Gamma(X(3872) \to \pi^+\pi^-\,J/\psi)} = 0.14 \pm 0.05$$

  - Angular analysis of $X(3872) \to \pi^+\pi^-\,J/\psi$ favors $J^P = 1^+$ Belle hep-ex/0505038

Isospin (G parity) is necessarily broken: $G(J/\psi\,\pi^+\pi^-) = -1$, $G(J/\psi\,\pi^+\pi^-\pi^0) = +1$. Cannot be a pure $\bar{c}c$.

The pion-system invariant mass distributions hint at $X(3872) \to J/\psi\rho \to J/\psi\,\pi^+\pi^-$ and $X(3872) \to J/\psi\omega \to J/\psi\,\pi^+\pi^-\pi^0$.

- It is highly likely that the X(3872) is an S-wave $D^0\bar{D}^{0*} + D^{0*}\bar{D}^0$ molecule, $J^{PC} = 1^{++}$. The isospin is badly broken, since the D^+D^{*-} pair is 8 MeV heavier.

If this interpretation is correct, the decay $X(3872) \to J/\psi\,\pi^0\pi^0$ is forbidden.

$$\frac{\Gamma(X(3872) \to J/\psi\,\pi^0\pi^0)}{\Gamma(X(3872) \to J/\psi\,\pi^+\pi^-)} < 1.3\,\frac{\Gamma(\psi' \to J/\psi\,\pi^0\pi^0)}{\Gamma(\psi' \to J/\psi\,\pi^+\pi^-)} \qquad \text{Belle '04}$$

Other views on X(3872) (and Molecular Quarkonium in general)

- Swanson's model of channel mixing

Quark exchange between the channels $D\bar{D}^*,\ \bar{D}D^*,\ \omega J/\psi,\ \rho J\psi$ PLB588, 189 (2004)

Successfully predicted $X(3872) \to \omega\, J/\psi$ at about the same rate as $X(3872) \to \omega\, J/\psi$.

- Maiani-Piccinini-Polosa-Riquer 'constituent' diquark model

Color-antisymmetric diquarks, e.g. $[cq]$ are constituents of 4-quark mesons.
X(3872) ($J^{PC} = 1^{++}$) is then (dominantly)$[cu]_{S=1} \times [\bar{c}\bar{u}]_{S=0} + [cu]_{S=0} \times [\bar{c}\bar{u}]_{S=1}$
The model predicts also $X_d = [cd][\bar{c}\bar{d}]$ resonance $M(X_d) - M(X_{(u)}) \approx 8 \pm 3\,\text{MeV}$, and also charged partners $X^{\pm}$. None of these have been seen (so far?). PRD71, 014028 (2005)

- Karliner-Lipkin non-relativistic model with color correlations hep-ph/0601193

$$H = \sum_i \frac{p_i^2}{2m_i} + \sum_{i \neq j} \frac{V_0}{4} \cdot \lambda_c^i \cdot \lambda_c^j \cdot r_{ij}^2$$

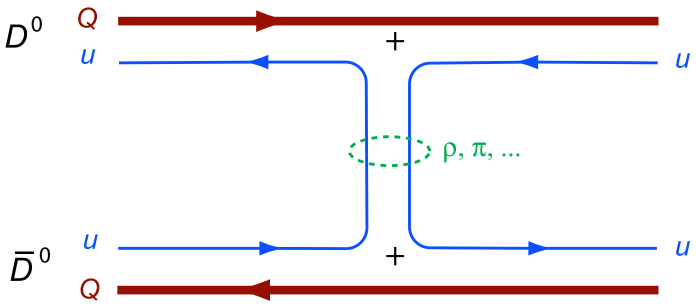
Without the kinetic term the model is known (Nambu '66) to give no multiquark binding. With the kin term: $M(cq\bar{c}\bar{q})$ is slightly larger than $2M(c\bar{q})$. However spin-dependent forces might explain $X(3872)$.
The 4-quark binding in this model has to occur in $bq\bar{c}\bar{q}$ and in $bq\bar{b}\bar{q}$ systems.

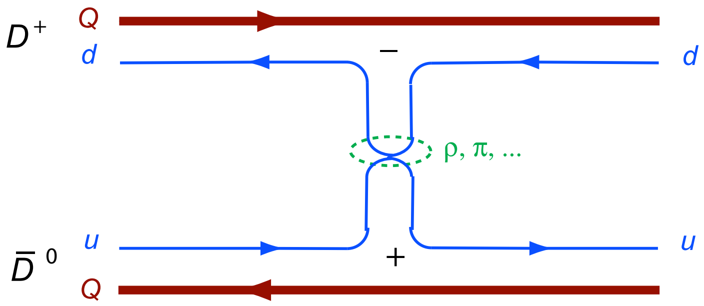
- Braaten-Kusunoki 'extreme $D\bar{D}^*$' model

X(3872) is only a soft $D\bar{D}^*$ pair. Conversely the soft dynamics of $D\bar{D}^*$ is 'universally' determined by the resonance pole.
Predicted $\mathcal{B}(B^0 \to K^0 X)/\mathcal{B}(B^+ \to K^+ X) \ll 1$ PRD71, 074005 (2005)
in possible contradiction with BaBaR's data.

Existence of X(3872) as a $D^0\bar{D}^{0*} + D^{0*}\bar{D}^0$ molecule does not necessarily imply existence of charged companions and/or of $D^+\bar{D}^{-*} + D^{+*}\bar{D}^-$.

- $D^0\bar{D}^{0*} + D^{0*}\bar{D}^0$ vs. charged, e.g. $D^+\bar{D}^{0*} + D^{+*}\bar{D}^0$ if $I = 1$ exchange is important:





- Mixing between $D^0\bar{D}^{0*} + D^{0*}\bar{D}^0$ and $D^+\bar{D}^{-*} + D^{+*}\bar{D}^-$ pushes the $D^0\bar{D}^{0*} + D^{0*}\bar{D}^0$ down and $D^+\bar{D}^{-*} + D^{+*}\bar{D}^-$ up.

- If the (dominantly) $D^+\bar{D}^{-*} + D^{+*}\bar{D}^-$ state exists as a resonance it is likely to be broad due to the decay $(D^+\bar{D}^{-*} + D^{+*}\bar{D}^-) \rightarrow D^0\bar{D}^{0*} + D^{0*}\bar{D}^0$.

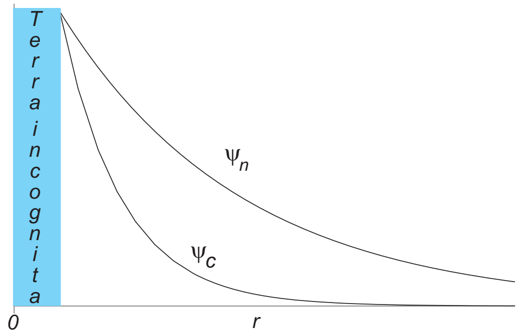
Generic picture (Fock decomposition)

One does not necessarily have to choose ‘either or’.

$$|X\rangle = a_0\,|D^0\bar{D}^{0*} + D^{0*}\bar{D}^0\rangle + a_1\,|D^+\bar{D}^{-*} + D^{+*}\bar{D}^-\rangle + \sum\,|\text{‘other'}\rangle$$

“$D^0\bar{D}^{0*} + D^{0*}\bar{D}^0$ molecule” $=$ the weight $|a_0|^2$ is a dominant part of 1. This is quantitative issue which can be tested experimentally.

The periphery of X(3872) should be $D^0\bar{D}^{0*} + D^{0*}\bar{D}^0$ ($\psi_n(r)$) with an admixture of $D^+\bar{D}^{-*} + D^{+*}\bar{D}^-$ ($\psi_c(r)$) at shorter distances.



$$\psi_n(r) = C_n\,\frac{\exp(-\kappa_n r)}{r}, \qquad\qquad \psi_c(r) = C_c\,\frac{\exp(-\kappa_c r)}{r}$$

$\kappa_n = \sqrt{2\,m_r\,w} = 44\,MeV\,\sqrt{w/MeV}$ with w= binding energy for $D^0\bar{D}^{0*}$;
$\kappa_c = \sqrt{2\,m_r\,(w + 8\,MeV)} \approx 125\,MeV$.

If isospin symmetry at short distances $\Rightarrow C_c \approx C_n \Rightarrow$
$|\langle X|D^+\bar{D}^{-*} + D^{+*}\bar{D}^-\rangle|^2/|\langle X|D^0\bar{D}^{0*} + D^{0*}\bar{D}^0\rangle|^2 \approx \kappa_n/\kappa_c$

- “Other” = the states at short distances, the ‘core’ of X(3872).

$J^{PC} = 1^{++}$ are the quantum numbers of a ${}^3P_1$ charmonium. Simplest assumption: ‘core’ $\approx (c\bar{c})_{{}^3P_1}$.

X(3872) is produced in hard processes through its $I = 0$ ‘core’, similar to $\approx (c\bar{c})_{{}^3P_1}$.

- Production in $p\bar{p}$ similar to charmonium ($\psi(2S)$, CDF),
- Production in $B \to KX$ similar to charmonium,
- $B(B \to KX)/B(B \to K\psi') \approx |a_{c\bar{c}}|^2$

Exp:

$$\frac{B(B^+ \to K^+X)\, B(X \to J/\psi\pi^+\pi^-)}{B(B^+ \to K^+\psi')\, B(\psi' \to J/\psi\pi^+\pi^-)} = 0.063 \pm 0.014$$

(BaBaR ‘05: $B(X \to J/\psi\pi^+\pi^-) > 4.2\%$)

$\Rightarrow |a_{c\bar{c}}|^2 \sim$ few percent (similar to the weight of the core in deutron).

- Expected $B^0 \to XK^0 \approx B^+ \to XK^+$ (BaBaR)

- Spin selection rule.

The S wave $(D^0\bar{D}^{*0}+\bar{D}^0 D^{*0})$ state is unique in the sense that the total spin of the heavy $c\bar{c}$ pair is fixed: $S_{c\bar{c}}=1$.

No orbital motion (S wave): $\vec{J}=\vec{S}_{c\bar{c}}+\vec{S}_{u\bar{u}}$. However $S_{c\bar{c}}=0\otimes S_{u\bar{u}}=1$ (and $S_{c\bar{c}}=1\otimes S_{u\bar{u}}=0$) gives C=-1. Only $S_{c\bar{c}}=1 \otimes S_{u\bar{u}}=1$ results in C=+1. The total spin of the light quark pair is not ‘traceable’: can be flipped in the mixing with the “core” states. The heavy quark spin is conserved in the limit $m_Q\to\infty$.

$\Rightarrow$ Spin selection rule (valid up to $O(\Lambda_{QCD}/m_c)$): the states with $S_{c\bar{c}}=1$ dominate in the Fock sum for $X(3872)$.

Some applications of the spin selection rule.

Transitions $X(3872)\to(c\bar{c})_{S=0}+light\ hadrons$ should be suppressed. E.g. $X(3872)\to\eta_c\,\pi\pi$ should be suppressed.

Transitions $X(3872)\to(c\bar{c})_{S=1}+light\ hadrons$ are favored: e.g. the observed $X(3872)\to\pi^+\pi^- J/\psi$ and $X(3872)\to\pi^+\pi^-\pi^0 J/\psi$ and also (yet unobserved) $X(3872)\to\pi^0\,\chi_{cJ}$ $(\Gamma\propto(2J+1))$.

Expected e.g.

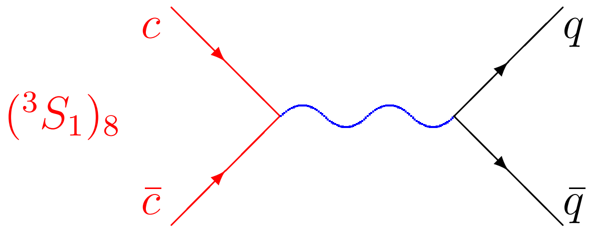
$$\frac{\Gamma(X\to\pi^0\,\chi_{c1})}{\Gamma(X\to\pi^+\pi^- J\psi)}\approx 0.35\left(\frac{0.5\,GeV}{\mu}\right)^2$$

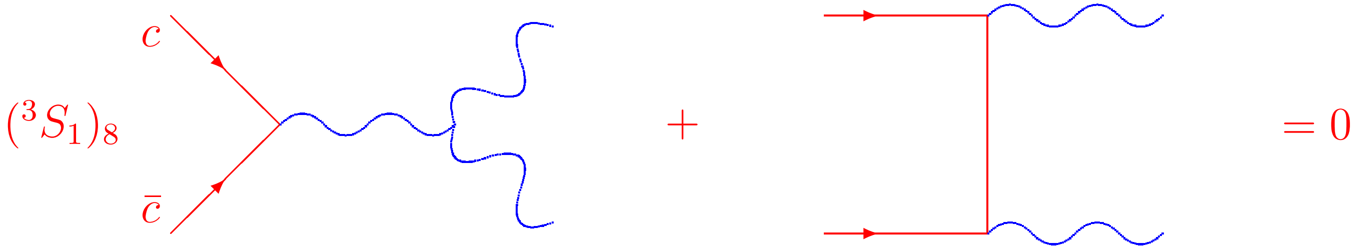
(μ = parameter for excitation of the $c\bar{c}$ P wave).

Can expect $\Gamma(X\to\pi^0\,\chi_{cJ})$ not much less than $\Gamma(X\to\pi^+\pi^- J/\psi)$.

- Decay $X(3872) \to$ *light hadrons.*

Annihilation of $c\bar{c}$. For $S_{c\bar{c}} = 1$ dominant mechanism is annihilation in ${}^3S_1$ color octet state. (“Charm burning.”)





Dominant: $c\bar{c} \to q\bar{q}$. ($({}^3S_1)_8 \to 2\,gluons$ vanishes at threhold.) (R.Barbieri et.al.’76, L.Okun & M.V. ’76)

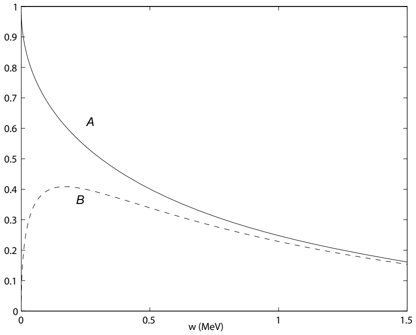
$$\Gamma(X \to \text{light hadrons}) \sim |a_{core}|^2 \alpha_s^2(m_c) \frac{(\mu')^3}{m_c^2} \sim O(100\, KeV)$$

$1/\mu'$: characteristic size of the $c\bar{c}$ in the core.

- Peripheral decays $X(3872) \to D^0 \bar{D}^0 \pi^0$, $X(3872) \to D^0 \bar{D}^0 \gamma$

Interference between $D^{0*} \to D^0 \pi^0(\gamma)$ and $\bar{D}^{0*} \to \bar{D}^0 \pi^0(\gamma)$. $(\Gamma(D^{*0} \to D^0 \pi^0) = 43 \pm 10\, KeV)$

$$\Gamma(X(3872) \to D^0 \bar{D}^0 \pi^0) = |a_0|^2 \Gamma(D^{*0} \to D^0 \pi^0)\, [A(w) + B(w)]$$

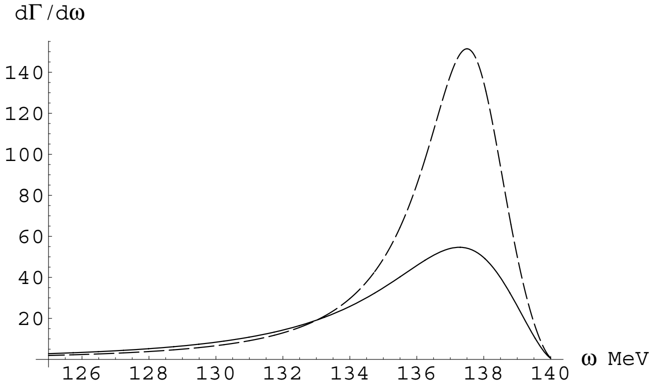


The current experimental status of $X(3872) \to D^0 \bar{D}^0 \pi^0$ is not clear.

$X \to D^0 \bar{D}^0 \gamma$:

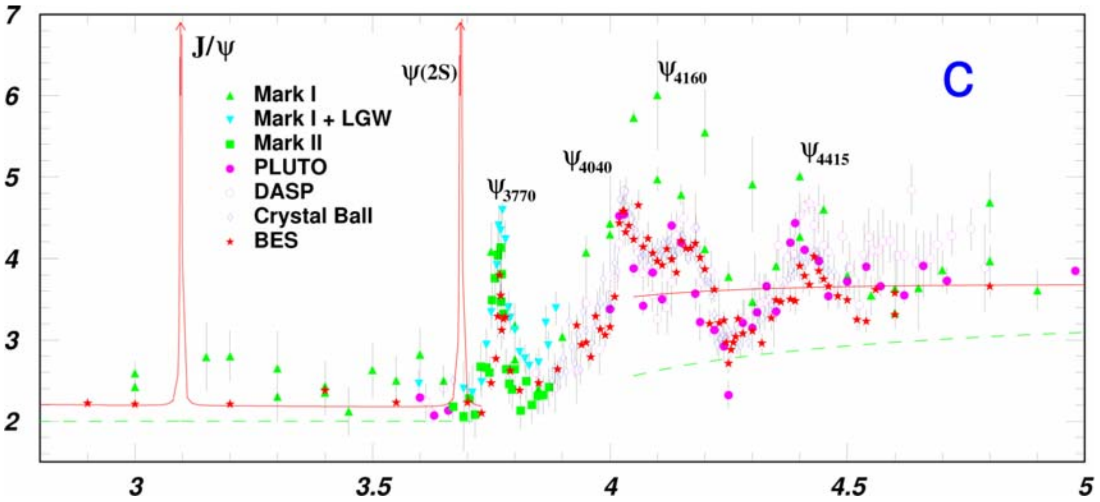
$$\Gamma(X \to D^0 \bar{D}^0 \gamma) = |a_0|^2\, \Gamma_\gamma \left(1 - \frac{2\,\kappa}{k_\gamma} \arctan \frac{k_\gamma}{2\,\kappa} \right)$$

$k_\gamma \approx 137\,\mathrm{MeV}$, $\Gamma_\gamma = \Gamma(D^{*0} \to D^0 \gamma) = 26 \pm 6\,\mathrm{KeV}$. Interference term: 0.32 at $w = 0.1\, MeV$, and 0.71 at $w = 0.5\, MeV$.



A small $X(3872) \to D^+ D^- \gamma$ is possible due to the charmonium “core” component: $X(3872) \to \psi(3770)\, \gamma \to D^+ D^- \gamma$.

- “$\psi(4040)$”



$\psi(4040) = D^*\bar{D}^*$ molecule?

At $\sqrt{s} \approx 4040\,\text{MeV}$, $R(D^*\bar{D}^*) \sim 1$ and the velocity $v \approx 0.1$. $R(D^*\bar{D}^*) = Q^2_{eff}\,\frac{3}{2}\,v^3$, $\Rightarrow Q_{eff} \approx 25 \div 30$.

The position of the resonance is not necessarily where the maximum of the cross section is: σ_{tot} is distorted by the p^3 onset of the $D^*\bar{D}^*$ channel. Can in fact be a bound “molecule” made of $D^*\bar{D}^*$, i.e. below 4020 MeV. Γ_{tot} is mostly due to coupling to lighter channels: $D^*\bar{D}$, $D_s\bar{D}_s$, $D\bar{D}$.

Important to scan the σ in detail for exclusive lighter channels.

- Reflection of the threshold $D^*\bar{D}^*$ resonance in lighter channels.

Denote: Heavy - $D^*\bar{D}^*$ channel, Light - one of the lighter charmed meson channels.

$W = E_{c.m.} - 2M(D^*)$.

Assume a resonance at $W = W_0 - \frac{i}{2}\,\Gamma_0$ strongly coupled to the H channel.

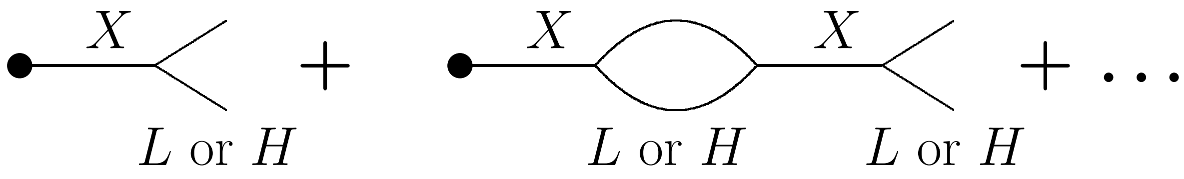
Resonance factor at $W > 0$

$$\mathcal{R} = \left[W - W_0 + \frac{i}{2}\left(\Gamma_0 + W\,\sqrt{\frac{W}{w}} \right) \right]^{-1}$$

The $W\,\sqrt{W/w}$ term is the extra absorbtion into the opening H channel.

At $W < 0$, $W = -\Delta$ from anlyticity, the resonance factor becomes

$$\mathcal{R} = \left[\frac{1}{2}\,\Delta\,\sqrt{\frac{\Delta}{w}} - \Delta - W_0 + \frac{i}{2}\,\Gamma_0 \right]^{-1}$$



The $\Delta^{3/2}$ term is more essential than the linear in Δ, at $\Delta \gtrsim w$ — becomes interesting if w is anomalously small (strong coupling to H).

If g is the coupling of the resonance to $D^*\bar{D}^*$, w is given by $w = g^{-4}\,M(D^*)^{-3}$. A value of w between 1 and 10 MeV would not be unusual.

$$A(e^+e^- \to L) = a_L + b_L\,\mathcal{R}$$

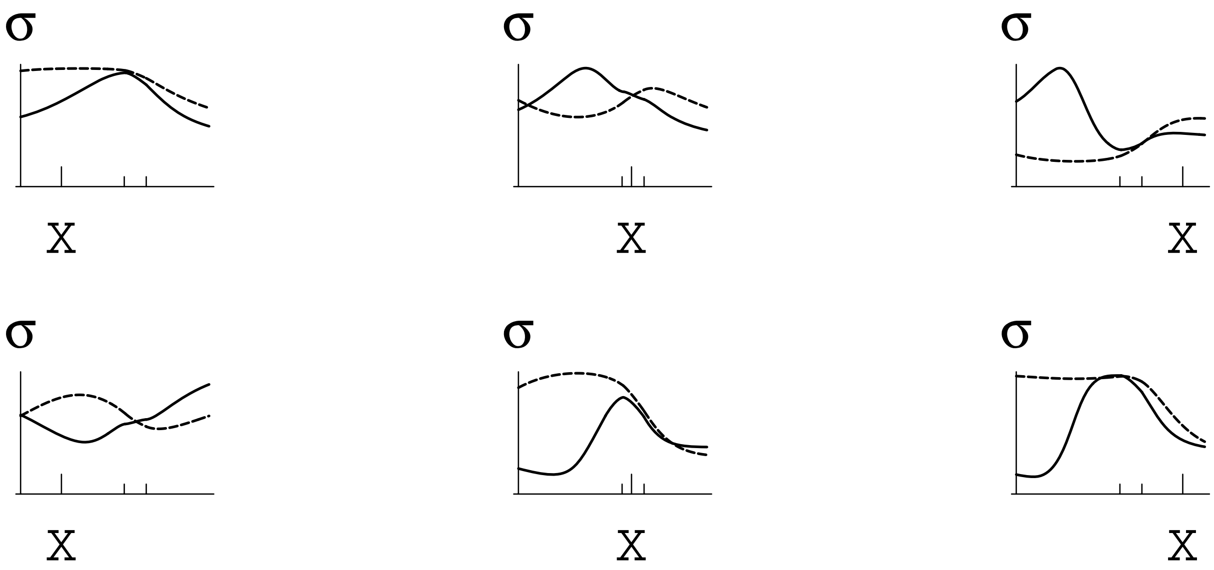


Figure 1: The types of behavior of the e^+e^- annihilation cross section (in arbitrary units) into one of the 'light' channels near the $D^* \bar{D}^*$ threshold. The horizontal axis in each plot spans the c.m. energy range from 3980 MeV to 4040 MeV. The $D^{*0}\bar{D}^{*0}$ and $D^{*+}\bar{D}^{*-}$ thresholds are shown with shorter vertical tick marks. The plots are shown for three assumed values of the 'nominal' position W_0 of the resonance X relative to the $D^{*0}\bar{D}^{*0}$ threshold: -20, 3, and 20 MeV, and the assumed value for this position is indicated in each plot by the longer vertical tick mark. In the plots in the upper row it is assumed that the coefficients a_L and b_L are relatively real and have the same sign. In the lower row of plots the relative sign of these coefficients is assumed to be negative. The solid lines in the plots correspond to $w = 1\,\text{MeV}$, and the dashed one are for $w = 10\,\text{MeV}$. The width parameter Γ_0 is fixed at $\Gamma_0 = 50\,\text{MeV}$.

- Angular distributions in $e^+e^- \rightarrow$ "$\psi(4040)$" $\rightarrow D^*\bar{D}^*$

Spin sturcture for $D^*\bar{D}^*$: $L = 1$ (P parity), $S = 0$ or $S = 2$ (C parity).

$$|\psi(4040)\rangle = \alpha\,|(D^*\bar{D}^*)_{S=2}\rangle + \beta|(D^*\bar{D}^*)_{S=0}\rangle + \text{“}non - (D^*\bar{D}^*)\text{”}$$

Angular distribution of $D^*\bar{D}^*$ in $e^+e^- \rightarrow \psi(4040) \rightarrow D^*\bar{D}^*$.

$$W(\theta) \propto 1 - \frac{|\alpha|^2 + 10\,|\beta|^2}{7\,|\alpha|^2 + 10|\beta|^2}\cos^2\theta$$

Pure $S = 2$: $1 - \frac{1}{7}\cos^2\theta$

Pure $S = 0$: $1 - \cos^2\theta$.

Distribution in the angle ϑ between D^* $(\bar{D}^*)$ and D $(\bar{D})$ from the decay $D^* \rightarrow D\pi$:

$$|\beta|^2 + \frac{1}{20}\,|\alpha|^2(13 + 21\cos^2\vartheta) + \frac{2}{\sqrt{5}}\,|\alpha|\,|\beta|\,\cos\varphi\,(3\,\cos^2\vartheta - 1)$$

Sensitive to the relative phase φ between α and β.

Distribution in the angle ϕ between the planes of the decays $D^* \rightarrow D\pi$ and $\bar{D}^* \rightarrow \bar{D}\pi$:

$$W(\phi) \propto 3|\alpha|^2 + (|\alpha|^2 + 10\,|\beta|^2)\,\cos^2\phi$$

Pure $S = 2$: $1 + \frac{1}{3}\cos^2\phi$

Pure $S = 0$: $\cos^2\phi$.

Other molecules (?)

- Generic nuclear-type forces ⇒ molecules should certainly exist at sufficiently heavy quark mass.

Is c ‘sufficiently’ heavy ?

Same conclusion in Karliner-Lipkin color correlation model.

- Maiani et.al. diquark-diquark hadrons (not exactly molecules) ⇒ should be more prolific than (appear to be) observed.
- Swanson model for other than X(3872) states ⇒ ?

Is X(3872) unique? In X(3872) a ‘coincidence’ of

- The possibility of a ‘long’ range pion exchange between D and $\bar{D}^*$.
- Significant ($\sim 8\,MeV$) splitting between the $D^0\bar{D}^{*0}$ and D^+D^{*-} thresholds. (Mixing ‘pushes’ the $D^0\bar{D}^{*0}$ down.)
- Proximity of the $J/\psi\,\rho$ and $J/\psi\,\omega$ thresholds. (Important in Swanson’s model.)

The tiny biding energy ($\lesssim 1\,MeV$) might require all these.

So far no definite prediction for other “molecular” charmonium or bottomonium or mixed (bc) states.

A study of “$\psi(4040)$” and possibly other thresholds might yield new insights.

Summary, Conclusions, . . .

- ‘Molecular’ states should (almost) inevitably exist at sufficiently heavy quark mass m_Q. Apriori it is not known whether the charmed quark is “sufficiently heavy”.
- The X(3872) is a likely candidate for a $(D^0 \bar{D}^{*0} + \bar{D}^0 D^{*0})$ $(J^{PC} = 1^{++})$ molecular system.
- The isospin is definitely broken in the X(3872) from the observation of both $X(3872) \to J/\psi\, \pi^+\pi^-$ and $X(3872) \to J/\psi\, \pi^+\pi^-\pi^0$.
- An observation of $X(3872) \to \chi_{c0}\, \pi^0$ would prove unnatural J^P.
- The molecular component of X(3872) can be probed in the decays $X(3872) \to D^0 \bar{D}^0 \pi^0$ and $X(3872) \to D^0 \bar{D}^0 \gamma$.
- Spin selection rule (if $J^{PC} = 1^{++}$): $X \to \chi_{cJ} \pi^0$ favored, $X \to \eta_c \pi\pi$ suppressed.
- The existence of other ‘molecules’ is still an open issue
- “$\psi(4040)$” is still a viable candidate for a molecule. A detailed scan of $e^+e^- \to D\bar{D}, D\bar{D}^*, D_s\bar{D}_s$ can be illuminatind. Also a study of angular correlations.

The heavy meson threshold(s) can be a treasure trove of very interesting hadronic physics.